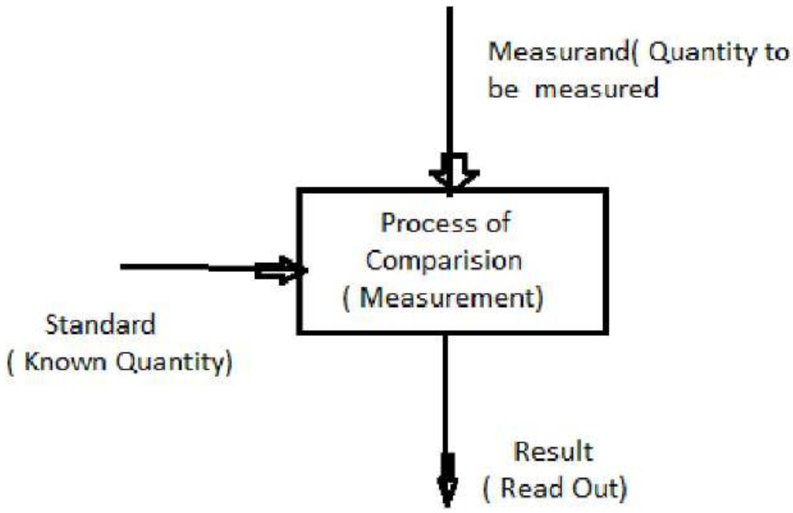
WHAT IS MEASUREMENT?

Measurement is the process of comparing unknown magnitude of certain parameter with the known predefined standard of that parameter. For instance, if we have to measure the temperature of the body, we measure it with the thermometer that has predefined scale indicating different values of the temperature. If we have to measure the length of the wall, we measure it with the measuring tape that has predefined markings on it. The measurement enables us obtaining magnitude of certain parameters whose value is not known by comparing them with the standards whose value is predefined.



PROCESS OF MEASUREMENT

INTRODUCTION & REQUIREMENTS OF MEASUREMENT-

For the measurements results to be accurate, two conditions should be met. Firstly, the standard which is used for comparison must be defined accurately and it should be universally accepted. For instance the weight cannot be just light or heavy. It should be light or heavy in comparison to some standard weight and should be measured accurately against it. The comparison of the unknown magnitude should be made with the recognized standard and it should produce some meaningful reading of the value.

The second important condition to be met for measurements is that the procedure applied for the measurements should be provable and there should be provable instruments for measurements. This means the methods for making the measurements and the instruments used for them should be reliable enough to make the correct measurements.

MEASUREMENT : A method to obtain information regarding the physical values of the variable.

- Definition —the quantitative comparison between a predefined standard and a measurand to produce a measured result

Measurement is the process of comparing unknown magnitude of certain parameter with the known predefined standard of that parameter.

GENERALIZED MEASUREMENT SYSTEM:-

In the direct method of measurement the physical quantity like length or mass are measured directly by the measuring instruments. The indirect method of measurement comprises of various stages for the measurement of the physical quantity like temperature, pressure, force etc, since they cannot be measured by the direct instruments. In this method the transducer is used which is connected to a host of other instruments to convert one form of energy that cannot be measured into the other form that can be measured easily. The input and the output values are calibrated so that for all the value of output the value of the input can be calculated.

The General System of Measurement comprises of three stages (see the fig1), these are:

1. **I) First stage - the detector-transducer stage.**

2. **II) Second stage – Intermediate modifying stage**

3. **III) Final stage – Terminating stage comprising of: indicator, recorder,some controller as individuals or in combination.**

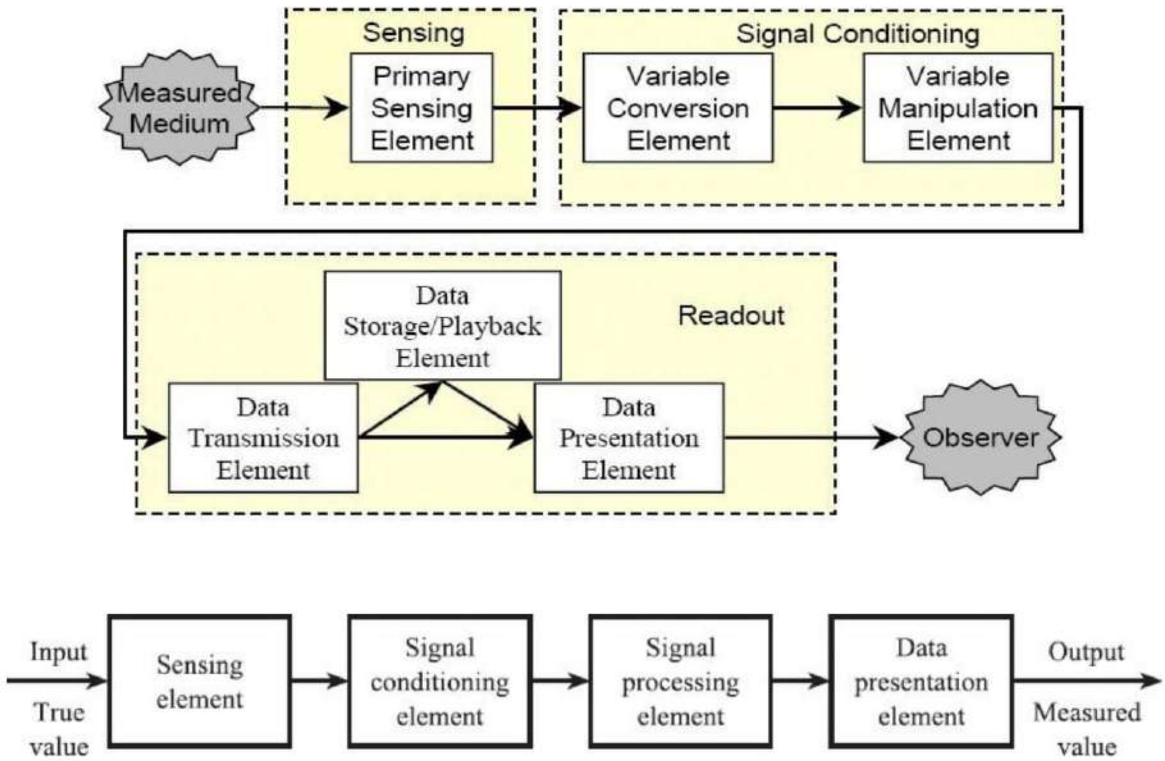


Fig.1 Generalized Measurement Systems

i) **Detector Unit: Primary Stage** : The main function of the first stage is to detect or sense the input signal. The input signal is usually the physical quantity that has to be measured. It can be temperature, pressure, force, velocity, or any other quantity that has to be measured. The devise most commonly used for detecting the input signal is the transducer or sensor. The transducer can detect the input quantity that has to be measured but cannot be measured directly by the instruments. It has the ability to convert these signals into analogous output that can be measured easily.

The detector, which is transducer, should be able to sense the input signals that are to be measured, but at the same time it should be insensitive to the other possible types of inputs.

For example if the pressure measuring transducer like bellow is being used, it should be sensitive to the pressure, but insensitive to the acceleration or other disturbances at the input. Similarly, if strain gauges are being used, they should be sensitive to temperature only. In actual practice it is very rare that the transducer is sensitive only to the signals that are to be measured.

II) Second Stage: Intermediate Modifying Stage

Let us continue with the second stage of general measurement system. The second stage or the intermediate modifying stage converts the input signal in the form that can be used easily by the final stage to indicate the value of the input physical quantity. The modifying stage may change the type of the input signal so that the output value can be measured easily. Or it may increase the amplitude and/or the power of the input signal to the level so as to drive the final terminating devices.

The intermediate modifying stage should be designed such that there is proper matching of characteristics between the first stage and second stage and the second stage and the final stage of measurement so that the output obtained is analogous to the input. The intermediate may also have to perform the functions of filtering the unnecessary input, and also integration, differentiation, telemetering etc, wherever required.

III) Final Stage: Terminating Stage (output stage)

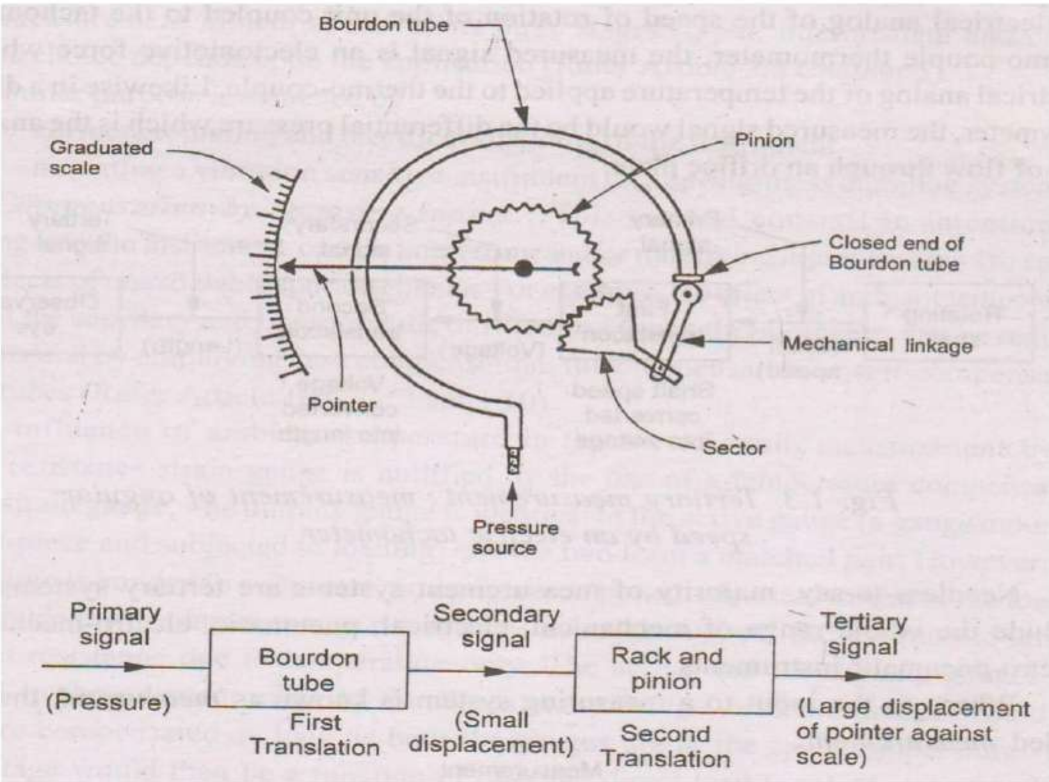
The final stage or the terminating stage provides the information about the input physical quantity in the form that can be easily read by the human beings or the controller. If the final output is intended to be recognized by the human beings it can be obtained in the form of the pointer movement on the predefined scale, in the digital format, by the graph etc. If the output is meant to perform some automatic function, it can be given to the controller that controls the operation of the next unit.

The output obtained at the final stage is calibrated against the input quantity so that the value shown by the pointer or the digital indicator is analogous to the input value. Thus the physical quantities like temperature, pressure, force etc which cannot be measured directly are measured with the help of the transducer in the form of pointer movement on the scale or the digital output.

EXAMPLE OF THE GENERAL MEASURMENT SYSTEM

Here the Bourdon tube senses the pressure, and it acts as the transducer that detects the quantity to be measured.

Let us come back to our previous example of measuring the pressure by using Bourdon tube. Here, the coiled oval shaped tube acts as the intermediate stage. When the pressure is applied at the inlet the oval tube tends to become circular, but inner and the outer diameters of the coil tend to remain the same. Due to this the coil tends to uncoil producing the angular motion of the tip of the coil, which is connected to the final stage of the pointer that indicates the value of the applied pressure.



The sealed tip of the coil is connected to the pointer via linkages and the gear arrangement. The pointer moves over the predefined scale that indicates the value of the pressure. When the pressure is applied the tip of the Bourdon tube uncoils, which moves the linkages and the gear that finally produce the rotary motion of the pointer on the scale indicating the actual value of the applied pressure.

APPLICATION OF MEASUREMENTS

Measurements are one of crucial parts of not only mechanical engineering but all types of engineering fields. Every branch of engineering involves two processes:

- **Monitoring and operation of process.**
- **Control of a process**.
- **Experimentation:**

- **Testing and performance operation**
- **Verification of properties or theory**
- **Information needed for analysis**

e.g. Checking or evaluation of:

Oil viscosity variation with temp.

Pump performance curve

piping head loss

Lift and drag of new airfoil shape…….etc.

Design, and operations and maintenance. The design may be machine design, building design, circuit design, transportation design, automobile design etc. The operations part involves operation of the machines, automobiles, various plants, circuits etc.

Both, the design, and operations and maintenance involve measurements. For instance while designing automobile we have to consider dimensions of various parts of the automobiles, the loads they can pick up etc. Similarly during the operations of the plant, say like industrial refrigeration plant, we have to measure parameters like pressure, temperature, etc.

In the power plant we have to measure

- Various quantities of the coal.
- The quantity of water in the boiler.
- The amount of steam produced along with its flow rate.
- Temperature and pressure.
- The amount of power produced.
- The outlet temperature of the steam from condenser etc.

Other field of application:

- Home
  - Thermometer
  - Barometer

    - Watch
- Road vehicles
    - speedometer
    - fuel gauge
- Industry
    - Automation
    - Process control
    - Boiler control

Types of Measurement

There are two methods of measurement:

1) Direct comparison with the standard

2) In-Direct comparison with the standard.

Both the methods are discussed below:

1.) Direct Comparison with the Standard

In the direct comparison method of measurement, we compare the quantity directly with the primary or secondary standard. Say for instance, if we have to measure the length of the bar, we will measure it with the help of the measuring tape or scale that acts as the secondary standard. Here we are comparing the quantity to be measured directly with the standard.

Even if you make the comparison directly with the secondary standard, it is not necessary for you to know the primary standard. The primary standards are the original standards made from certain standard values or formulas. The secondary standards are made from the primary standards, but most of the times we use secondary standards for comparison since it is not always feasible to use the primary standards from accuracy, reliability and cost point of view. There is no difference in the measured value of the quantity whether one is using the direct method by comparing with primary or secondary standard.

The direct comparison method of measurement is not always accurate. In above example of measuring the length, there is limited accuracy with which our eye can read the readings, which can be about 0.01 inch. Here the error does not occur because of the error in the standards, but because of the human limitations in noting the readings. Similarly, when we measure the mass of any body by comparing with some standard, it's very difficult to say that both the bodies are of exactly the same mass, for some difference between the two, no matter how small, is bound to occur. Thus, in direct method of measurement there is always some difference, however small, between the actual value of the quantity and the measured value of the quantity.

2) In-Direct Method of Measurement

There are number of quantities that cannot be measured directly by using some instrument. For instance we cannot measure the strain in the bar due to applied force directly. We may have to record the temperature and pressure in the deep depths of the ground or in some far off remote places. In such cases indirect methods of measurements are used.

In the indirect method of measurements some transducing devise, called transducer, is used, which is coupled to a chain of the connecting apparatus that forms the part of the measuring system. In this system the quantity which is to be measured (input) is converted into some other measurable quantity (output) by the transducer. The transducer used is such that the input and the output are proportional to each other. The readings obtained from the transducer are calibrated to as per the relations between the input and the output thus the reading obtained from the transducer is the actual value of the quantity to be measured. Such type of conversion is often necessary to make the desired information intelligible.

The indirect method of measurements comprises of the system that senses, converts, and finally presents an analogues output in the form of a displacement or chart. This analogues output can be in various forms and often it is necessary to amplify it to read it accurately and make the accurate reading of the quantity to be measured. The majority of the transducers convert mechanical input into analogues electrical output for processing, though there are transducers that convert mechanical input into analogues mechanical output that is measured easily.

STATIC & DYNAMIC CHARACTERISTICS OF INSTRUMENTS:

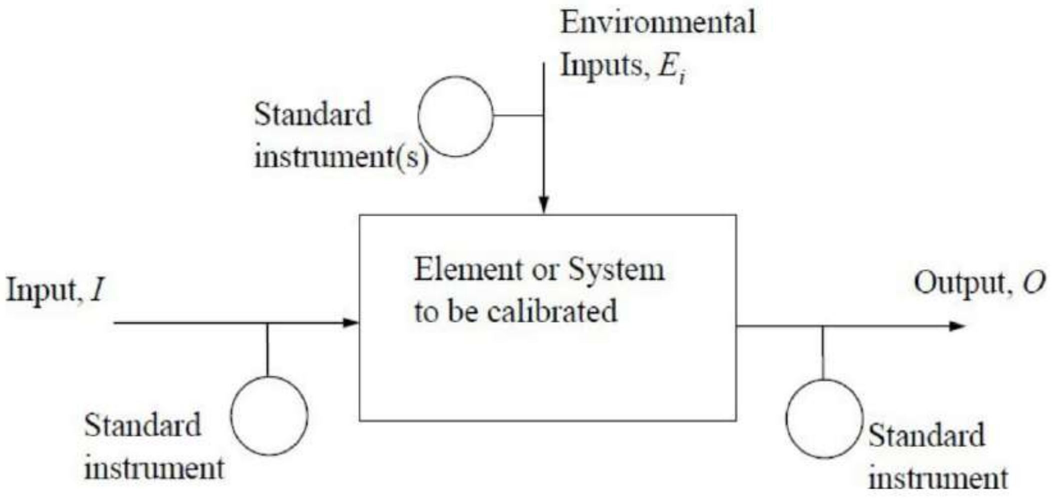
Characteristics of measurement systems

The system characteristics are to be known, to choose an instrument that most suited to a particular measurement application. The performance characteristics may be broadly divided into two groups, namely '**static**' and '**dynamic**' characteristics.

STATIC CHARACTERISTICS- the performance criteria for the measurement of quantities that remain constant, or vary only quite slowly.

DYNAMIC CHARACTERISTICS- the relationship between the system input and output when the measured quantity (measurand) is varying rapidly

calibration:



General diagram for calibration

STATIC AND DYNAMIC CHARACTERISTICS -.

- Instrument systems are usually built up from a serial linkage of distinguishable building blocks.
- The actual physical assembly may not appear to be so but it can be broken down into a representative diagram of connected blocks.
- The sensor is activated by an input physical parameter and provides an output signal to the next block that processes the signal into a more appropriate state.
- A fundamental characterization of a block is to develop a relationship between the input and output of the block.
- All signals have a time characteristic.
- It is essential to consider the behaviour of a block in terms of both the static and dynamic states.
- The behaviour of the static regime alone and the combined static and dynamic regime can be found through use of an appropriate mathematical model of each block.

CLASSIFICATION OF INSTRUMENTS

The classifications of instruments are given below

1. Automatic and manual instruments: -

The manual instruments require the services of an operator while the automatic types do not. For example, the temperature measurements by mercury-in-glass thermometer is automatic as the instrument indicates the temperature without requiring any manual assistance. However, the measurement of temperature by a resistance thermometer incorporating a wheatstone bridge in its circuit is manual in operation as it needs an operator for obtaining the null position.

Automatic instruments are proffered because of their fast dynamic response and low operational cost.

2. Self-generating and power-operated instruments:-

In a self-generating instrument, the output entirely or almost entirely by the input signal. The instrument Does not require any outside power in performing its function. For example, the motive power in a mercury-in-glass thermometer is supplied wholly by the thermal expansion of mercury and as such it constitutes a self-operated device. Likewise, the dial indicator is an active device since all the energy required to operate the instrument is furnished by the system whose displacement is being measured.

Some other common examples of active instruments are:-

exposure meter of a camera which is essentially a photo voltaic cell.

- bourdon gauge for the measurement of pressure.
- pitot-tube for the measurement of velocity.
- tacho-generator for rotational speed measurement.

Some instruments require some auxiliary source of power such as compressed air, electricity, hydraulic supply etc. For there operation and hence are called externally powers instruments. In these devices, the input signal supplies only an insignificant portion of the output power. For example, in the electro-mechanical measurement system an exciter has been incorporated to supply electric energy for the transducer. Likewise, the digital revolution counter is a passive device. The power to drive the solenoid comes from the a.c. power lines and not from the rotating shaft.

Some other common examples of passive instruments are:-

- · lvdt used in the measurement of displacement, force, pressure.
- · voltage-dividing potentiometer which converts rotation or displacement into potential difference.
- · photo-conductive transducer which translate light information to resistance information.
- · strain-gauge load cell using wheat stone bridge circuit.
- · resistance thermometers and thermisters.

3. Self-contained and remote indicating instruments: -

The different elements of a self-contained instrument are contained in one physical assembly. In a remote indicating instrument, the primary sensing element may be located at a sufficiently long distance from the secondary indicating element. In the modern instrumentation technology, there is a trend to install remote indicating instruments where the important indications can be displayed in the central control rooms.

4. Deflection and null output instruments:-

In null-type instruments, the physically effect caused by the quantity being measured in nullified by generating an equivalent opposing effect. The equivalent null causing effect then provides a measure of the unknown quantity.

A deflection type instrument is that in which the physical effect generated by the measuring quantity is noted and correlated to the measurand.

A distinction between the null and deflection mode of operation can be made by considering the working of a pan balance and a platform scale. In the pan balance the unknown weight is placed of one pan of the balance and weights of known value are placed in the other pan until a balance condition is indicated by zero or null position of the pointer.

In a platform scale, i.e. the deflection mode, the weight of an object placed on the platform of the scale is indicated by the relative displacement between the pointer and a dial.

The null-type devices are slow in operation, have poor dynamic response but are more accurate and sensitive, and do not interfere with the state of the quantity being measured. Deflection instruments are simple in construction and operation, have good dynamic response. However, they interfere with the state of measurand and as such do not determine its exact state/value/condition.

5. Analog and digital instruments: -

The signals of an analog unit vary in a continuous fashion and can take on infinite number values in a given range. Wrist watch, speedometer of an automobile, fuel gauge, ammeters and voltmeters are examples of analog instruments.

Signals varying in discrete steps and taking on a finite number of different values in a given range are digital signals and the corresponding instruments are of digital type. For example, the timers on a scoreboard, the calibrated balance of a platform scale, and odometer of an automobile are digital instruments.

The digital instruments convert a measured analog voltage into digital quantity which is displayed numerically, usually by neon indicator tubes. The output may either be a digit for every successive increment of the input or be a coded discrete signal representative of the numerical value of the input. The digital devices have the advantage of high accuracy high speed and the elimination of human operational errors. However, these instruments are unable to indicate the quantity which is a part of the step value of the instrument. The importance of the digital instrumentation is increasing very fast due to the applications of the digital computers for data handling, reduction and in automatic controls. Apparently it becomes necessary to have both analog-to-digital converters at input to the computers and digital-to-analog converters at the output of the computers.

STANDARDS & ITS TYPES:

Standards are the fundamental reference for a system of weights and measures, against which all other **measuring** devices are compared.**standards are subdivided into following four categories or grades:**

1. Primary Standards (Reference Standards).
2. Secondary Standards (Calibration Standards).
3. Tertiary Standards (Inspection Standards).
4. Working Standards (Workshop Measuring Standards).

1. Primary Standards (Master Standards):

The primary standard is also known as Master Standard, and is preserved under the most careful conditions. These standards are not commonly in use. They are used only after long internals. They solely used for comparing the secondary standards. Sometimes it is also called Reference Standards.

2. Secondary Standards (Calibration Standards):

The secondary standard is more or less similar to the primary standard. They are nearly close in accuracy with primary standards. The secondary standard is compared at regular intervals with primary Stands and records their deviation. These Standards are distributed to a number of places where they are kept under safe custody. They are used occasionally for comparing the territory standards.

3. Tertiary Standards (Inspection Standards):

The Tertiary standard is the first standard to be used for reference purpose in workshops and laboratories. They are used for comparing the working standards. These are not used as frequently and commonly as the working standards but more frequency than secondary standards. Tertiary standards should also be maintained as a reference for comparison at intervals for working standards.

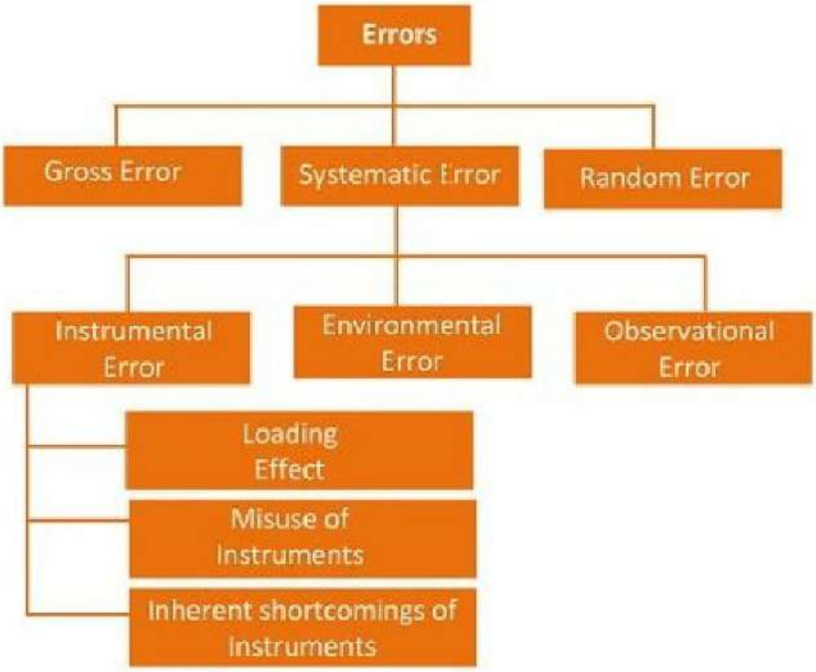
4. Working Standards (Workshop Measuring Standards):

The working standard is used for actual measurement in workshop or laboratories by the workers. These standards should also be as accurate as possible to the tertiary standard. But sometimes, lower grades of materials can be used for their manufacturing to reduce cost.

TYPES OF ERRORS IN MEASUREMENT

The error may arise from the different source and are usually classified into the following types. These types are

1. **Gross Errors**
2. **Systematic Errors**
3. **Random Errors**



Their types are explained below in details.

1. Gross Errors

The gross error occurs because of the human mistakes. For examples consider the person using the instruments takes the wrong reading, or they can record the incorrect data. Such type of error comes under the gross error. The gross error can only be avoided by taking the reading carefully.

For example – The experimenter reads the 31.5ºC reading while the actual reading is 21.5Cº. This happens because of the oversights. The experimenter takes the wrong reading and because of which the error occurs in the measurement.

Such type of error is very common in the measurement. The complete elimination of such type of error is not possible. Some of the gross error easily detected by the experimenter but some of them are difficult to find. Two methods can remove the gross error.

Two methods can remove the gross error. These methods are

- The reading should be taken very carefully.
- Two or more readings should be taken of the measurement quantity. The readings are taken by the different experimenter and at a different point for removing the error.

2. Systematic Errors

The systematic errors are mainly classified into three categories.

1. Instrumental Errors
2. Environmental Errors
3. Observational Errors

2 (i) Instrumental Errors

These errors mainly arise due to the three main reasons.

(a) Inherent Shortcomings of Instruments – Such types of errors are inbuilt in instruments because of their mechanical structure. They may be due to manufacturing, calibration or operation of the device. These errors may cause the error to read too low or too high.

For example – If the instrument uses the weak spring then it gives the high value of measuring quantity. The error occurs in the instrument because of the friction or hysteresis loss.

(b) Misuse of Instrument – The error occurs in the instrument because of the fault of the operator. A good instrument used in an unintelligent way may give an enormous result.

For example – the misuse of the instrument may cause the failure to adjust the zero of instruments, poor initial adjustment, using lead to too high resistance. These improper practices may not cause permanent damage to the instrument, but all the same, they cause errors.

(c) Loading Effect – It is the most common type of error which is caused by the instrument in measurement work. For example, when the voltmeter is connected to the high resistance circuit it gives a misleading reading, and when it is connected to the low resistance circuit, it gives the dependable reading. This means the voltmeter has a loading effect on the circuit.

The error caused by the loading effect can be overcome by using the meters intelligently. For example, when measuring a low resistance by the ammeter-voltmeter method, a voltmeter having a very high value of resistance should be used.

2 (ii) Environmental Errors

These errors are due to the external condition of the measuring devices. Such types of errors mainly occur due to the effect of temperature, pressure, humidity, dust, vibration or because of the magnetic or electrostatic field. The corrective measures employed to eliminate or to reduce these undesirable effects are

- The arrangement should be made to keep the conditions as constant as possible.
- Using the equipment which is free from these effects.
- By using the techniques which eliminate the effect of these disturbances.
- By applying the computed corrections.

2 (iii) Observational Errors

Such types of errors are due to the wrong observation of the reading. There are many sources of observational error. For example, the pointer of a voltmeter resets slightly above the surface of the scale. Thus an error **occurs** (because of parallax) unless the line of vision of the observer is exactly above the pointer. To minimise the parallax error highly accurate meters are provided with mirrored scales.

3. Random Errors

The error which is caused by the sudden change in the atmospheric condition, such type of error is called random error. These types of error remain even after the removal of the systematic error. Hence such type of error is also called residual error.

3.5.1. Statistical Averages

When several multi-sample observations are taken experimentally there is a scatter of the data about some central value, one of the methods, presenting the result in the form of a histogram (also called a *frequency distribution curve*). Thus, histogram (Fig. 3.1) is a pictorial representation of the test data where the ordinates indicate the number of observed readings of a particular value. With more and more data taken at smaller and smaller increments, the histogram finally changes into a *smooth curve*. The smooth curve is symmetrical with respect to the central value.

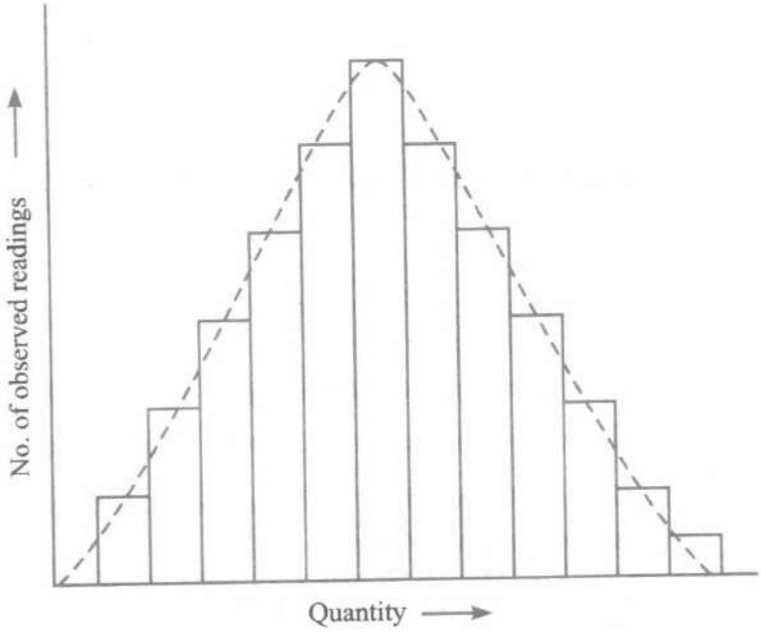


Fig. 3.1. Histogram.

(*i*) **Arithmetic mean (AM).** *The most probable value of measured variable (variate) is the arithmetic mean of the number of readings taken.* We arrive at the best approximation if the number of readings of the same quantity are very large. The mean is computed by summing all the values and dividing by the number of measurements. If $q_1, q_2, q_3 \ldots q_n$ be the set of measured values of a *quantity*, then the arithmetic mean $\bar{q}$ will be given by,

$$\bar{q} = \frac{q_1 + q_2 + q_3 + \ldots + q_n}{n} = \frac{1}{n}\sum_{i=1}^{n} q_i \qquad \ldots(3.4)$$

(*ii*) **Geometric mean (G.M.).** It is defined as the nth root of the product of *n* terms.

$$\text{Geometric mean (G.M.)} = \sqrt[n]{q_1 \cdot q_2 \cdot q_3 \ldots\ldots q_n} \qquad \ldots(3.5)$$

(*iii*) **Median.** *The middle value of a set of an "odd" number of readings, if variables are arranged in numerical order is called the median.* For an "*even*" number of readings, the median is the *arithmetic average of the two central readings.*

(*iv*) **Mode.** *The value of the variable that occurs most frequently is termed as mode.*

- For a symmetrical distribution curve, the three measures of centre value *viz.* '*mean*', '*median*' and '*mode*' *are equal.*

3.5.2. Dispersion from Mean

The property which denotes the extent to which the values are dispersed about the central value is termed as **dispersion** *or* **stored** *or* **scatter**.

A large dispersion indicates that some factors involved in the measurement process are not under close control and therefore it becomes difficult to estimate the measured quantity with confidence and definiteness.

The simplest possible measure of dispersion is the **Range** *which is the difference between greatest and least values of data.*

(*i*) **Deviation.** *The departure of the observed reading from the arithmetic mean of the group of readings is termed as deviation.* Let the deviation of reading q_1 be d_1 and that of reading q_2 be d_2 etc.,

Then,
$$d_1 = q_1 - \bar{q}$$
$$d_2 = q_2 - \bar{q}$$
$$\cdots \quad \cdots\cdots$$
$$d_n = q_n - \bar{q} \qquad \text{...(3.6)}$$

and,
$$\bar{q} = \frac{\Sigma(q_n - d_n)}{n} \qquad \text{...(3.7)}$$

$$\begin{aligned}\text{Algebraic sum of deviations} &= d_1 + d_2 + d_3 + \ldots + d_n \\ &= (q_1 - \bar{q}) + (q_2 - \bar{q}) + (q_3 - \bar{q}) + \ldots (q_n - \bar{q}) \\ &= (q_1 + q_2 + q_3 + \ldots + q_n) - n\bar{q} = 0\end{aligned}$$

$$(\because q_1 + q_2 + q_3 + \ldots + q_n) = n\bar{q}$$

Thus, the *algebraic sum of deviations is zero.*

The above mathematical result implies that the deviations can be *positive as well as negative.*

(*ii*) **Average deviation.** *It is defined as the sum of the absolute values of deviations divided by the number of readings.*

Average deviation may be expressed as

$$\bar{d} = \frac{|d_1| + |d_2| + |d_3| + \ldots\ldots + |d_n|}{n}$$

$$= \frac{\Sigma |d|}{n}, \quad \text{or,} \quad \sum_{i=1}^{n} \frac{|q_i - \bar{q}|}{n} \qquad \text{...(3.8)}$$

The average deviation is taken as a measure of accuracy of measurement.

- The average deviation is an *indication of the precision of the instruments* used in making the measurements. *Highly precise instruments yield a low average deviation between readings.*

(*iii*) **Standard deviation.** "*Standard deviation*" (or "*mean square deviation*") of an infinite number of data is defined *as the square root of the sum of the individual deviations squared divided by the number of readings.*

Thus, standard deviation,

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + ... + d_n^2}{n}}$$

$$= \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\sum_{i=1}^{n} \frac{(q_i - \bar{q})^2}{n}} \qquad ...(3.9)$$

- In practice, however the number of observations is *finite*. When the number of observations is *greater than 20*, standard deviation is denoted by the symbol **σ** while if it is *less than 20* the symbol used is **s**. The standard devitation of a finite number of data is given by :

$$s = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + ... d_n^2}{n-1}} = \sqrt{\frac{\Sigma d^2}{n-1}} \qquad ...(3.10)$$

Variance. *The square of the standard deviation is called* **variance.**

Variance, $V = (\text{Standard deviation})^2$

$$= \sigma^2 = \frac{d_1^2 + d_2^2 + d_3^2 .... d_n^2}{n} = \frac{\Sigma d^2}{n} \qquad ...(3.11)$$

But, when the number of observation is less than 20,

variance, $$V = s^2 = \frac{\Sigma d^2}{n-1} \qquad ....(3.12)$$

3.5.3. Best Value from a Sample of Readings

Let, q_b = The best value (approximately the true value) from a set of readings/data.

Then, the deviation or individual component error,

$$d_i = q_i - q_b \qquad \text{where, } i = 1, 2, 3, ... n$$

This deviation may be $+ ve$, *zero* or $- ve$.

(*i*) **Sum of deviations :**

$$\sum_{i=1}^{n} d_i = \sum_{i=1}^{n} (q_i - q_b) \qquad ....(3.13)$$

In case of a *best value*, the sum of deviations is equal to *zero*.

i.e., $$\sum_{i=1}^{n} (q_i - q_b) = 0 \qquad ...(3.14)$$

or, $$q_b = \frac{1}{n} \sum_{i=1}^{n} q_i$$, which is the arithmetic mean.

Thus the arithmetic mean may be interpreted as the best value around which the chances of error being $+ ve$ or $- ve$ likely to be equal.

(*ii*) $$d_i = q_i - q_b$$

Squaring both sides, we have:

$$d_i^2 = q_i^2 - 2q_i\, q_b + q_b^2$$

Sum of the square of deviations,

$$\sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} q_i^2 - 2q_b \sum_{i=1}^{n} q_i + nq_b^2 \qquad \text{...(3.15)}$$

The sum of the square of deviations, corresponding to the *best value* would be *minimum*. This condition can be obtained by setting the deviation w.r.t, q_b equal to zero;

i.e.,

$$\frac{d}{dq_b}\left[\sum_{i=1}^{n} d_i^2\right] = 0 \qquad \text{...(3.16)}$$

or,

$$\frac{d}{dq_b}\left[\sum_{i=1}^{n} q_i^2 - 2q_b \sum_{i=1}^{n} q_i + nq_b^2\right] = 0$$

As all the q_i^2 *s* are constant, the differentiation gives:

$$-2\sum_{i=1}^{n} q_i + 2nq_b = 0$$

or,

$$q_b = \frac{1}{n}\sum_{i=1}^{n} q_i \qquad \text{...(3.17)}$$

which is again the arithmetic mean. Obviously, *the arithmetic mean is the most probable value or the best value which one can obtain from a set of scattered measurement data.*

Example 3.9

By using a micrometer screw the following readings were taken of a certain physical length:

1.34, 1.38, 1.56, 1.47, 1.42, 1.44, 1.53, 1.48, 1.40, 1.59 mm

Assuming that only random errors are present, calculate the following:

(i) Arithmetic mean,

(ii) Average deviation,

(iii) Standard deviation, and

(iv) Variance.

Solution. *Given* : Reading of the micrometer screw = 1.34, 1.38, 1.56, 1.47, 1.42, 1.44, 1.53, 1.48, 1.40, 1.59 mm.

(*i*) **Arithmetic mean :** We know that, arithmetic mean,

$$\bar{q} = \frac{1}{n}\sum_{i=1}^{n} q_i \qquad \text{...(Eqn. 3.1)}$$

$$= \frac{q_1 + q_2 + q_3 + q_4 + q_5 + q_6 + q_7 + q_8 + q_9 + q_{10}}{10}$$

$$= \frac{1.34 + 1.38 + 1.56 + 1.47 + 1.42 + 1.44 + 1.53 + 1.48 + 1.40 + 1.59}{10}$$

= **1.461 mm** **(Ans.)**

(*ii*) **Average deviation :** From eqn. (3.6), the *deviations* are :

$$d_1 = q_1 - \overline{q} = 1.34 - 1.461 = -0.121$$

$$d_2 = q_2 - \overline{q} = 1.38 - 1.461 = -0.081$$

$$d_3 = q_3 - \overline{q} = 1.56 - 1.461 = +0.099$$

$$d_4 = q_4 - \overline{q} = 1.47 - 1.461 = +0.009$$

$$d_5 = q_5 - \overline{q} = 1.42 - 1.461 = -0.041$$

$$d_6 = q_6 - \overline{q} = 1.44 - 1.461 = -0.021$$

$$d_7 = q_7 - \overline{q} = 1.53 - 1.461 = +0.069$$

$$d_8 = q_8 - \overline{q} = 1.48 - 1.461 = +0.019$$

$$d_9 = q_9 - \overline{q} = 1.40 - 1.461 = -0.061$$

$$d_{10} = q_{10} - \overline{q} = 1.59 - 1.461 = 0.129$$

∴ *Average deviation,*

$$\overline{d} = \frac{\Sigma|d|}{n}\left(\text{or } \sum_{i=1}^{n}\frac{|q_i - \overline{q}|}{n}\right)$$

$$= \frac{0.121 + 0.081 + 0.099 + 0.009 + 0.041 + 0.021 + 0.069 + 0.019 + 0.061 + 0.129}{10}$$

$$= \mathbf{0.065} \quad \textbf{(Ans.)}$$

It may be noted that for calculation of average deviations, the *signs of deviations are not considered.*

(*iii*) **Standard deviation :** Since the number of readings is 10, which is less than 20, therefore equation (3.10) is used for calculation of standard deviation.

$$\text{Standard deviations, } s = \sqrt{\frac{\Sigma d^2}{n-1}} \qquad \text{...[Eqn. (3.10)]}$$

$$\text{or, } s = \sqrt{\frac{(-0.121)^2 + (-0.081)^2 + (+0.099)^2 + (0.009)^2 + (-0.041)^2 + (-0.021)^2 + (+0.069)^2 + (0.019)^2 + (-0.061)^2 + (0.129)^2}{(10-1)}}$$

$$= \sqrt{\frac{0.05869}{9}} = \mathbf{0.0807\ mm} \quad \textbf{(Ans.)}$$

(*iv*) **Variance :**

$$\textit{Variance}, V = s^2 = (0.0807)^2 = \mathbf{0.00651\ mm^2} \quad \textbf{(Ans.)}$$

Example 3.10

Eight different students turned in the circuit for resonance and the values of resonant frequency in kHz were recorded as :

412, 428, 423, 415, 426, 411, 423, 416

Calculate the following :

(i) Arithmetic mean,

(ii) Average deviation,

(iii) Standard deviation, and

(iv) Variance,

Solution. *Given :* Values of resonant frequency : 412, 428, 423, 415, 426, 411, 423, 416

(*i*) **Arithmetic mean**

Arithmetic mean,

$$\bar{q} = \frac{\Sigma q}{n} = \frac{412 + 428 + 423 + 415 + 426 + 411 + 423 + 416}{8}$$

$$= \mathbf{419.25\ kHz}\ \ (\textbf{Ans.})$$

(*ii*) **Average deviation :** The deviations are :

$$d_1 = q_1 - \bar{q} = 412 - 419.25 = -7.25 \text{ kHz}$$

$$d_2 = q_2 - \bar{q} = 428 - 419.25 = +8.75 \text{ kHz}$$

$$d_3 = q_3 - \bar{q} = 423 - 419.25 = 3.75 \text{ kHz}$$

$$d_4 = q_4 - \bar{q} = 415 - 419.25 = -4.25 \text{ kHz}$$

$$d_5 = q_5 - \bar{q} = 426 - 419.25 = +6.75 \text{ kHz}$$

$$d_6 = q_6 - \bar{q} = 411 - 419.25 = -8.25 \text{ kHz}$$

$$d_7 = q_7 - \bar{q} = 423 - 419.25 = +3.75 \text{ kHz}$$

$$d_8 = q_8 - \bar{q} = 416 - 419.25 = -3.25 \text{ kHz}$$

∴ *Average deviation,*

$$\bar{d} = \frac{\Sigma |d|}{n}$$

$$= \frac{7.25 + 8.75 + 3.75 + 4.25 + 6.75 + 8.25 + 3.75 + 3.25}{8}$$

$$= \mathbf{5.75\ kHz}\ \ (\textbf{Ans.})$$

(For the calculation of average deviation, the *signs of deviations are not considered*)

(*iii*) **Standard deviation :** Since the number of readings is 8, which is less than 20, therefore eqn. (3.10) is used for calculation of standard deviation.

$$\text{Standard deviation, } s = \sqrt{\frac{\Sigma d^2}{n-1}} \qquad \text{...[Eqn (3.10)]}$$

$$\text{or, } s = \sqrt{\frac{(-7.25)^2 + (+8.75)^2 + (+3.75)^2 + (-4.25)^2 + (+6.75)^2 + (-8.25)^2 + (+3.75)^2 + (-3.25)^2}{(8-1)}}$$

$$= \mathbf{6.54\ kHz}\ (\textbf{Ans.})$$

(*iv*) **Variance:**

$$\textit{Variance, } \mathbf{V} = s^2 = (6.54)^2 = \mathbf{42.77\ (kHz)^2}\ \ (\textbf{Ans.})$$

3.5.4. Normal or Gaussian Curve of Errors

In experimental work the random effects are normally studied with reference to the Gaussian or normal error distribution. Here, the following *assumptions* are made :

(*i*) All observations include a large number of small random disturbing effects, and these small random errors may be both +*ve* and –*ve*. Thus, there is a strong central tendency, *i.e., there is a greater probability of small errors and large errors of both +ve and – ve values.*

(*ii*) There is equal probability of +*ve* and –*ve* disturbing effects. As such the probability curve of errors plotted against the magnitude of error will be *symmetrical about the zero value.*

The **law of probability** *states that the normal occurrence of deviations from average value of an infinite number of measurements or observations can be expressed as:*

$$P = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2} \qquad ...(3.18)$$

where, P = Number of readings at any deviation '*d*', *i.e.*, probability of occurrence of deviation *d*,

σ = Standard deviation, and

x = Magnitude of deviation from mean.

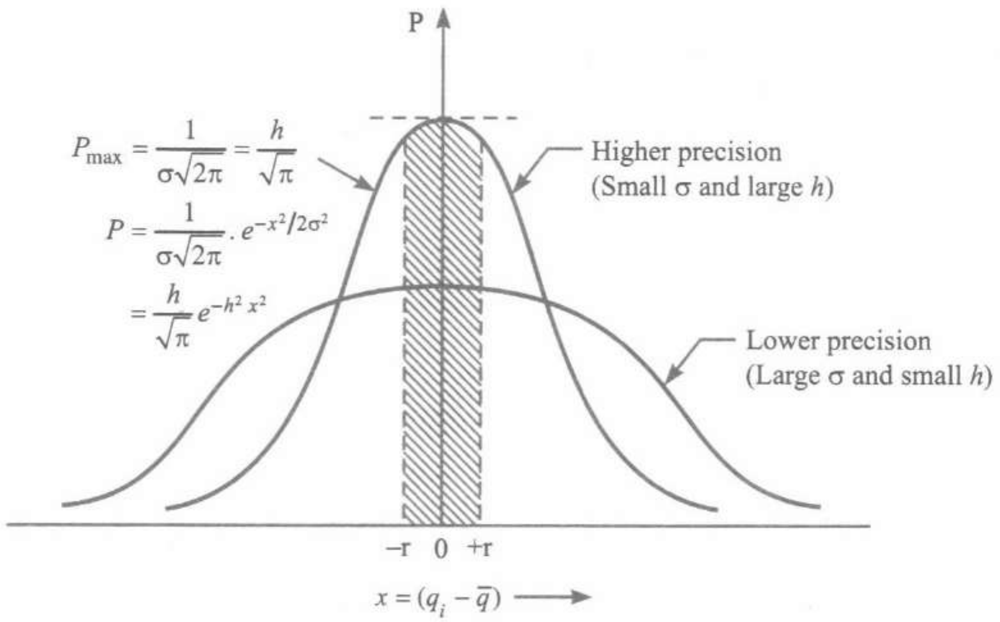


Fig. 3.2. Gaussian distribution curve.

Fig. 3.2. shows a Gaussian distribution curve in which P has been plotted as a function of x. The curve is symmetrical about the mean value and may be regarded as the limiting form of a histogram. The two values of standard deviation σ_1 and σ_2 ($\sigma_1 > \sigma_2$) have been illustrated in the given figure. *Larger the value of σ, the flatter the curve and hence larger the expected error of all measurements.*

If we integrate P between the limits $x = \pm x$, we shall obtain the area under the curve between these two values of x. Because the curve is related to the probability that the measurand lies within same range, we have

$$\int_{-x}^{+x} P\, dx = 1.0 \qquad ...(3.19)$$

i.e., area under the probability curve is *unity*.

If the total number of readings are represented by unity, then the fraction of the total number of readings lying between x_1 and x_2 will be represented by the area under the curve between these two values of deviation;

i.e.,
$$n_{1-2} = \int_{x_1}^{x_2} P\,dx$$

If the area between x_1 and x_2 is 0.6, then 60 per cent of the deviations fall between x_1 and x_2.

Precision index. It may be observed from the normal distribution function that the maximum probability occurs when deviation is *zero*, *i.e.*, $q = \bar{q}$ and the corresponding value of probability is,

$$P_{max} = \frac{1}{\sigma\sqrt{2\pi}} = \frac{h}{\sqrt{\pi}} \qquad ...(3.20)$$

where, $h = \dfrac{1}{\sigma\sqrt{2}}$ is called the **precision index**.

A large value of h represents high precision of the data because the probability of occurrence of variates in a given range falls off *rapidly* as the deviation increases because the variates tend to cluster (become closer) into a narrow range. On other hand, a small value of h represents low precision because the probability of occurrences of variates in a given range falls off *gradually* as the deviation increases; this is because the variates are spread over a wide range.

Probable error : Let us consider the two points $-r$ and $+r$ marked in Fig. 3.2. These points are so located that the area bounded by the curve, the X-axis and the ordinates erected at $x = -r$ and $x = +r$ is equal to half of the total area under the curve. That is half the deviations lie between $x = \pm r$. The location of the points r can be worked out as follows:

$$\int_{-r}^{r} P\,.\,dx = \frac{1}{2}$$

or,
$$\frac{h}{\sqrt{\pi}} \int_{-r}^{r} e^{-h^2x^2}\,dx = \frac{1}{2}$$

This gives *probable error*,

$$r = \frac{0.4769}{h} \qquad ...(3.21)$$

Average deviation for the normal curve: In case of normal curve,

Average deviation,

$$\bar{d} = \int_{-\infty}^{+\infty} |x|\,P\,dx$$

($|x|$ represents the numerical value of deviation)

$$= \frac{h}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-h^2x^2}\,x.dx = \frac{2h}{\sqrt{\pi}} \int_{0}^{+\infty} e^{-h^2x^2}\,x.dx$$

$$= \frac{2h}{\sqrt{\pi}} \int_{0}^{+\infty} e^{-h^2z}$$

$$= \frac{2h}{\sqrt{\pi}} \times \frac{1}{2}\left[\frac{e^{-h^2 z}}{-h^2}\right]_0^{+\infty} \qquad \left[\begin{aligned}&\text{where, } z = x^2\\ &dz = 2x.dx\\ &x.dx = \frac{dz}{2}, \text{ and}\\ &\int e^{ax}dx = \frac{e^{ax}}{a}\end{aligned}\right]$$

$$= \frac{h}{\sqrt{\pi}}\left[\frac{0-1}{-h^2}\right] = \frac{h}{\sqrt{\pi}} \times \frac{1}{h^2} = \frac{1}{h\sqrt{\pi}}$$

$$= \frac{r}{0.4769\sqrt{\pi}} = \frac{r}{0.8453}$$

i.e.,

$$\bar{d} = \frac{r}{0.8453} \qquad ...(3.22)$$

Standard deviation for the normal curve: We know that, standard deviation σ is given by :

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}} \qquad ....[\text{Eqn. } (3.9)]$$

Following a method similar to that followed above, we have:

$$\sigma^2 = \frac{2h}{\sqrt{\pi}} \int_0^{+\infty} e^{(-h^2x^2)}\, x^2.dx = \frac{1}{2h^2}$$

or,

$$\sigma = \frac{1}{\sqrt{2}h} \qquad ...(3.23)$$

$$= \frac{r}{0.4769 \times \sqrt{2}}$$

$$= \frac{r}{0.6745} \qquad ...(3.24)$$

From eqns. (3.22) and (3.24), we have *probable error,*

$$r = 0.8453\, \bar{d} \qquad ...(3.25)$$

and,

$$r = 0.6745\, \sigma \qquad ...(3.26)$$

Also, *Standard devitation of mean*

$$\sigma_m = \frac{\sigma}{\sqrt{n}} \qquad ...(3.27)$$

and, *Standard deviation of standard deviation,*

$$\sigma_\sigma = \frac{\sigma}{\sqrt{2n}} = \frac{\sigma_m}{\sqrt{2}} \qquad ...(3.28)$$

3.5.5. Probability Tables

The probability of an individual measurement to fall within the deviations x_1 and x_2 from the mean is given by:

$$P(x_1, x_2) = \frac{h}{\sqrt{\pi}} \int_{x_1}^{x_2} e^{(-h^2x^2)}\, dx$$

for a Normal Gaussian curve.

The parameter $h = \dfrac{1}{\sigma\sqrt{2}}$ and the mean value $\overline{q}$ may be known from the raw data. Since the evaluation of the integral is a cumbersome and a tedious task, hence **probability tables** have been prepared from which the value of probability can be directly read off.

Let $$y = \sqrt{2}hx = \frac{x}{\sigma},$$

or, $$x = \sigma.y$$

Now, $$P(x_1, x_2) = P(y_1, y_2)$$

$$= \frac{1}{\sqrt{2\pi}} \int_{y_1}^{y_2} e^{(-y^2/2)}\, dy \qquad ...(3.29)$$

where, $$y_1 = \sqrt{2}hx_1$$

and, $$y_2 = \sqrt{2}\, h\, x_2$$

Eqn. (3.29) may be written as:

$$P(y_1, y_2) = \frac{1}{\sqrt{2\pi}} \int_{0}^{y_2} (e)^{(-y^2/2)}\, dy - \frac{1}{\sqrt{2\pi}} \int_{0}^{y_1} e^{(-y^2/2)}\, .dy \qquad ...(3.30)$$

Each of the two integrals in equation (3.30) is a function of a single parameter y_1 or y_2. Therefore, probability,

$$P(0, y) = \frac{1}{\sqrt{2\pi}} \int_{0}^{y} e^{(-y^2/2)}\, dy \qquad ...(3.31)$$

The results of this integral are given in Table 3.1 and can be used to compute the proability between the limits x_1 and x_2. Tabulated values of the integral represent the area under the normalised Gaussian error curve for $y = 0$ and $y = y$ as shown in Fig. 3.3.

As the curve is *symmetrical* the same table may be used for – ve deviations replacing y by $-y$ in equation 3.31 since the value of proability is not affected by this substitution.

— Suppose we are required to find the probability that the deviation lies between $\pm\, \sigma$. For the + ve deviation σ,

$$y = \frac{q - \overline{q}}{\sigma} = \frac{\sigma}{\sigma} = 1$$

The table 3.1 indicates the probability value $P = 0.3413$ for $y = 1.0$. Due to symmetrical nature of the error curve, an equal probability would exist for the deviations to lie between $-\sigma$ and 0. Hence the probability that deviation lies between $\pm\,\sigma$ works out to be $2 \times 0.3413 = 0.6826$.

TABLE 3.1 Probability Tables

y	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3079	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4430	0.4441
1.6	0.4452	0.4463	0.4474	0.4485	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4700	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4762	0.4767
2.0	0.4773	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4865	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4980	0.4980	0.4981
2.9	0.4981	0.4982	0.4983	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.5000	0.5000	0.5000

- Further, let it be required to find the probable error which represents the interval about the mean in which lies half of the data. This corresponds to symmetrical area about the mean which has a value of 0.5. Only half of this area lies to the right of $y = 0$ and accordingly we look for the value of y in the probability table that corresponds to *0.25*. The corresponding value of $y =$ **0.6745** and accordingly the probable error is given by : *Probable error* $= \pm\, 0.6745\,\sigma$.

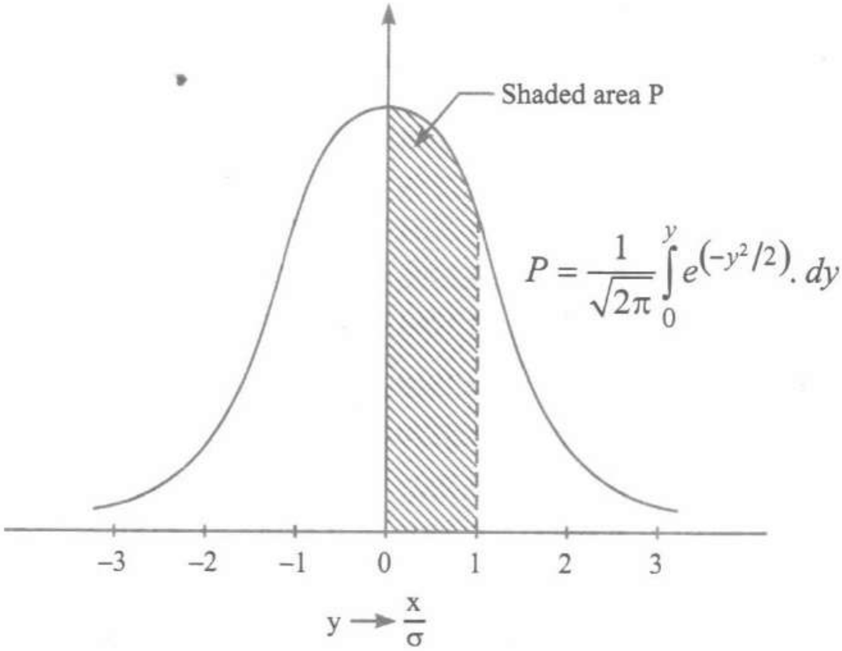


Fig. 3.3. Graphical representation of probability.

Example 3.11

While measuring a temperature the following ten readings were recorded :

39.6, 39.9, 39.7, 39.9, 40.0, 39.8, 39.9, 39.8, 40.4 and 39.7°C.

Calculate the following :

(i) The mean,

(ii) The standard deviation,

(iii) The probable error of one reading,

(iv) The probable error of mean, and

(v) The range.

Solution. The statistical values of the given data are computed with the help of the following table;

Observation number	Temperature reading, °C (q)	Deviation, $d = q - \bar{q}$	d^2
1	39.6	− 0.27	0.0729
2	39.9	+ 0.03	0.0009
3	39.7	− 0.17	0.0289
4	39.9	+ 0.03	0.0009
5	40.0	+ 0.13	0.0169
6	39.8	− 0.07	0.0049

(Contd...)

Observation number	Temperature reading, °C (q)	Deviation, $d = q - \bar{q}$	d^2
7	39.9	+ 0.03	0.0009
8	39.8	− 0.07	0.0049
9	40.4	+ 0.53	0.2809
10	39.7	− 0.17	0.0289
$n = 10$	$\Sigma q = 398.7$ $\bar{q} = \frac{\Sigma q}{n} = \frac{398.7}{10} = 39.87$		$\Sigma d^2 = 0.441$

(*i*) The mean value of the reading :

$$\bar{q} = \frac{\Sigma q}{n} = \frac{398.7}{10} = \mathbf{39.87°C} \quad \textbf{(Ans.)}$$

(*ii*) The standard deviation :

$$s = \sqrt{\frac{\Sigma d^2}{(n-1)}}$$ ... since the number of observations is 10 (*i.e.*, less than 20).

$$= \sqrt{\frac{0.441}{10-1}} = \mathbf{0.22°C} \textbf{ (Ans.)}$$

(*iii*) Probable error of one reading :

$$r_1 = 0.6745\ \mathrm{s} = 0.6745 \times 0.22 = \mathbf{0.15°C} \quad \textbf{(Ans.)}$$

(*iv*) Probable error of mean:

$$r_m = \frac{r_1}{\sqrt{n-1}} = \frac{0.15}{\sqrt{10-1}} = \mathbf{0.05°C} \quad \textbf{(Ans.)}$$

(*v*) Range :

$$Range = 40.4 - 39.6 = \mathbf{0.8°C} \textbf{ (Ans.)}$$

Example 3.12

In a test temperature, measurements of temperature were made 100 times with variations in apparatus and procedures. After applying the corrections the following results were obtained :

Temperature °C	197	198	199	200	201	202	203	204	205
Frequency of Occurrence	2	4	10	24	36	14	5	3	2

Calculate the following :

(i) Arithmetic mean,

(ii) Average deviation,

(iii) Standard deviation,

(iv) Variance,

(v) Probable error of one reading,

(vi) Probable error of the mean, and

(vii) Standard deviation of the deviation.

Solution. The computations in tabular form are given below :

Temperature °C T	Frequency of occurrence, f	$T \times f$	Deviation $d\left(=T-\frac{\Sigma T \times f}{n}\right)$	$f \times d$	d^2	$f \times d^2$
197	2	394	– 3.77	– 7.54	14.213	28.426
198	4	792	– 2.77	– 11.08	7.673	30.692
199	10	1990	– 1.77	– 17.7	3.133	31.330
200	24	4800	– 0.77	– 18.48	0.593	14.232
201	36	7236	+ 0.23	+ 8.28	0.053	1.908
202	14	2828	+ 1.23	+ 17.22	1.513	21.182
203	5	1015	+ 2.23	+ 11.15	4.973	24.865
204	3	612	+ 3.23	+ 9.69	10.433	31.299
205	2	410	+ 4.23	+ 8.46	17.893	35.786
Total	$n = 100$	$\Sigma T \times f =$ 20077		$\Sigma\lvert f \times d\rvert$ = 109.6		$\Sigma f \times d^2$ = 219.72

(*i*) Arithmetic mean :

$$\text{Arithmetic mean} = \frac{\Sigma T \times f}{n} = \frac{20077}{100} = \mathbf{200.77°C} \quad \text{(Ans.)}$$

(*ii*) Average deviation :

$$\sigma = \frac{\Sigma\,|\,f \times d\,|}{n} = \frac{109.6}{100} = \mathbf{1.096°C} \quad \text{(Ans.)}$$

(*iii*) Standard deviation :

Standard deviation, $\boldsymbol{\sigma} = \sqrt{\frac{\Sigma f \times d^2}{n}} = \sqrt{\frac{219.72}{100}} = \mathbf{1.482°C}$ **(Ans.)**

(*iv*) Variance :

$$\text{Variance, } V = \sigma^2 = (1.482)^2 = \mathbf{2.196°C} \quad \text{(Ans.)}$$

(*v*) Probable error of one reading :

Probable error of one reading,

$$r_1 = 0.6745\sigma = 0.6745 \times 1.482 \simeq \mathbf{1°C} \quad \textbf{(Ans.)}$$

(*vi*) Probable error of the mean :

$$r_m = \frac{r_1}{\sqrt{n}} = \frac{1(°C)}{\sqrt{100}} = \mathbf{0.1°C} \quad \text{(Ans.)}$$

(*vii*) Standard deviation of the standard deviation :

Standard deviation of the mean,

$$\sigma_m = \frac{\sigma}{\sqrt{n}} = \frac{1.482}{\sqrt{100}} = 0.148°C$$

$\therefore$ Standard deviation of the standard deviation,

$$\sigma_\sigma = \frac{\sigma_m}{\sqrt{2}} = \frac{0.148}{\sqrt{2}} = \mathbf{0.1046°C} \quad \text{(Ans.)}$$

Example 3.13

A known current of 80A is measured by an ammeter. If 40% of the readings are within 0.8 A of true value, determine the following :

(i) The standard deviation for the meter.

(ii) The probability of an error of 1.2 A.

Solution. **(*i*) Standard deviation for the meter :**

Considering a normal distribution curve, half the readings, *i.e.*, 20% of the total readings have a +*ve* error and the remaining 20% have a –*ve* error.

From probability tables, it is found that corresponding to a probability of $P(y) = 0.2$,

$$y = \frac{x}{\sigma} = 0.5248$$

∴ Standard deviation,

$$\sigma = \frac{x}{y} = \frac{0.8}{0.5248} = \mathbf{1.524} \text{ (Ans.)}$$

(*ii*) **Probability of an error of 1.2 A :**

Now, $$y = \frac{x}{\sigma} = \frac{1.2}{1.524} = 0.787$$

Corresponding to $y = 0.787$, probability is 0.2842

∴ Probability of an error of 1.2 A = 2 × 0.2842 = 0.5684

Thus about **57% of the readings** are *within ± 1.2 A of the true value.* **(Ans.)**

Example 3.14

An underdamped galvanometer was energized 100 times under the same carefully controlled experimental conditions and maximum deflection was read in each case. The readings were distributed about a mean value of 21.9 mm and had a probable error of 2.1 mm. How many of 100 readings would you estimate exceeded 25 mm ?

Solution. Deviation, $x = 25 - 21.9 = 3.1$ mm

Probable error, $r = 2.1$ mm

From eqn. 3.24, standard deviation,

$$\sigma = \frac{r}{0.6745} = \frac{2.1}{0.6745} = 3.1$$

$$\text{Ratio, } y = \frac{x}{\sigma} = \frac{3.1}{3.1} = 1.0$$

Corresponding to ratio $y = \frac{x}{\sigma} = 1.0$, the area under the curve is 0.3413 (from tables 3.1).

∴ No. of readings having a deviation within ± 3.1 mm

$$= 2 \times 0.3413 \times 100 \simeq 68$$

No. of readings exceeding a deviation of ± 3.1 mm

$$= 100 - 68 = 32$$

Hence, the number of readings having a deviation of + 3.1 mm

$$= \frac{32}{2} = 16$$

Thus the *number of readings exceeding a maximum deflection of 25 mm i.e., (21.9 + 3.1)* mm is **16** (**Ans.**)

Example 3.15

A machine shop was assigned the task to manufacture 10,000 aluminium rods of a nominal length of 20 mm; the rod length was stipulated neither to exceed 20.25 mm and nor to be smaller than 19.5 mm. When inspected for quality control, it was found that 1000 of the rods were too long to fit into a gauge set at 20.25 mm. Determine the number of remaining 9000 rods which will conform to the specifications. Assume that the measurement data conforms to Gaussian normal distribution curve.

Solution. In case of normal distribution, there is equal probability of the +*ve* and –*ve* errors about the mean. Therefore 5000 rods have lengths greater than 20 mm and 5000 rods have lengths smaller than 20 mm. Since 1000 rods have length greater than 20.25 mm, therefore,

The number of rods where length lies between 20 mm and 20.25 mm

$$= 5000 - 1000 = 4000$$

∴ Probability that 4000 rods have a value greater than 20 mm and less than 20.25 mm,

$$P(y) = \frac{4000}{10,000} = 0.4$$

From probability tables, it is found that corresponding to a probability of $P(y) = 0.4$,

$$y = \frac{x}{\sigma} \simeq 1.3 \qquad \text{....(From table 3.1)}$$

∴ Standard deviation,

$$\sigma = \frac{x}{1.3} = \frac{20.25 - 20.00}{1.3} = \frac{0.25}{1.3} = 0.1923$$

Now, for rods with nominal length of 19.5 mm and 20 mm,

$$y = \frac{x}{\sigma} = \frac{(20 - 19.5)}{0.1923} = 2.60$$

Corresponding probability, $P(y) = 0.4953$ ....(From table 3.1)

∴ Number of rods that have lengths between 19.5 mm and 20 mm

$$= 10000 \times 0.4953 = 4953$$

Hence, *total number of rods whose lengths lie between specified limits of 19.5 mm to 20.25 mm.*

$$= 4000 + 4953 = \mathbf{8953} \text{ (Ans.)}$$

Example 3.16

The speed of a hydraulic turbine model, which is running at 1500 rpm, is measured by a tachometer. The hydraulic turbine is subject to variations in speed. For a sample of 20 readings at this speed, how many readings would you expect between 1485 and 1515. Assume that the tachometer gives a normal set of deviations with precision index of 0.04 at 1500 r.p.m.

Solution. *Given :* Deviation, $d = q - \overline{q} = \pm 15$ r.p.m.

and, precision index, $h = 0.04$

Standard deviation, $\sigma = \frac{1}{\sqrt{2}\,h}$ ...[Eqn. (3.23)]

$$= \frac{1}{\sqrt{2} \times 0.04} = 17.68$$

$$y = \frac{x}{\sigma} = \frac{q-\bar{q}}{\sigma} = \frac{15}{17.68} = 0.8484$$

Corresponding to $y = 0.8484$, as read from the probability tables

$$P(y) = 0.3015$$ ...(From table 3.1)

Probability of an error ± 15 r.p.m. = **2 × 0.3015 = 0.603**

Thus we expect 60% or 0.6 × 20 = **12 readings** *to lie between 1485 to 1515 r.p.m.* **(Ans.)**

Example 3.17

The depth of water which had a nominal value of 15 cm, was measured with a hook. Measurements were taken 40 times and 10 readings were found to lie outside a particular range. Assuming that the measurements conform to normal distribution with precision index 9 cm^{-1}, determine the prescribed range.

Solution. *Given :* Nominal value of depth of water = 15 cm

Total number of times the measurements taken = 40

No. of measurement readings found to lie outside a particular range = 10

Precision index, $h = 9$ cm^{-1}

Prescribe range : Probability of falling within a particular range

$$\doteq \frac{(40-10)}{40} = 0.75$$

Half of these measurements have a +*ve* error and half have a –*ve* error.

$$\therefore \quad P(y) = \frac{0.75}{2} = 0.375$$

From probability tables, it is found that corresponding to a probability

$$P(y) = 0.375,$$

$$y = \frac{x}{\sigma} = \frac{q-\bar{q}}{\sigma} = 1.15$$ ....(From table 3.1)

Now, standard deviation, $\sigma = \frac{1}{\sqrt{2}\,h}$ ...[Eqn. (3.23)]

$$= \frac{1}{\sqrt{2} \times 9} = 0.0786$$

$$\therefore \quad x = (q-\bar{q}) = 1.15 \times 0.0786 = 0.0904 \text{ cm}$$

Thus, 75 percent of depth measurements lie within the **range (15 ± 0.0904) cm.** **(Ans.)**

Example 3.18

(a) *A mass flow meter, using electromagnetic principle is set to record a large number of readings. The fixed mass flow rate is 100 kg/s; half of the readings lie between 104.8 and 95.2 kg/s. The sample measurements give a straight line between the (0, 50%) points on the probability coordinates. Calculate the precision index of the instrument.*

(b) *An alarm rings when the flow meter gives a reading of 88 kg/s.*

(i) *How many false alarms are expected in the month of November if the flow is checked automatically 4 times a day ?*

(ii) *What will be the value of precision index if one wants to reduce the false alarm rate by a factor of 2 ?*

Solution. (*a*) The overall probability is 0.5. Considering normal distribution, 0.25 of the measurements have a +*ve* error and the rest 0.25 of the measurements have a –*ve* error.

Corresponding to a probability of $P(y) = 0.25$, from probability tables, we have:

$$y = \frac{x}{\sigma} = 0.675 \qquad \text{...(From table 3.1)}$$

∴ Standard deviation,
$$\sigma = \frac{x}{0.675} = \frac{4.8}{0.675} = 7.11$$

Precision index,
$$h = \frac{1}{\sqrt{2}\,\sigma} = \frac{1}{\sqrt{2} \times 7.11} = \mathbf{0.099} \quad \textbf{(Ans.)}$$

(*b*) (*i*) For a mass flow rate of 88 kg/s,

$$y = \frac{x}{\sigma} = \frac{100-88}{7.11} = 1.69$$

Corresponding probability $P(y) = 0.45$ ...(From table 3.1)

Thus, it is expected that $(0.5 - 0.45) = 0.05$ or 5% of the flow measurements will fall in the range of false alarms.

Number of measurements in the month of November $= 30 \times 4 = 120$

∴ *Expected false alarms* $= 120 \times 0.05 = \mathbf{6}$ **(Ans.)**

(*ii*) Reduced number of false alarms $= \dfrac{1}{2} \times 6 = 3$

Probability of false alarms $= \dfrac{3}{120} = 0.025$ (or 2.5%)

Probability of data to lie in the tolerant band,

$$P(y) = 0.5 - 0.025 = 0.475$$

Corresponding to $P(y) = 0.475$

$$y = \left(\frac{x}{\sigma}\right) = 1.96 \qquad \text{...(From table 3.1)}$$

or, Standard deviation, $\sigma = \frac{x}{1.96} = \frac{100-88}{1.96} = 6.12$

$\therefore$ *Precision index,* $h = \frac{1}{\sqrt{2}\,\sigma} = \frac{1}{\sqrt{2} \times 6.12} = \mathbf{0.1155}$ **(Ans.)**

3.5.6. Odds

Odds *is the number of chances that a particular reading will occur when error limit is specified*. Thus the probability of occurrence can be stated in terms of *odds.*

Example. When the error limits are specified as $\pm 0.6745\sigma$, the chances are that 50 percent of observations will lie between the above limits or in other words we can say odds are 1 to 1.

The odds can be calculated as under:

$$\text{Probability of occurrence} = \frac{\text{Odds}}{\text{Odds}+1} \qquad ...(3.32)$$

Certain deviations, their probability of occurrence and the corresponding odds are given in the table 3.2 below.

TABLE 3.2 Deviations, Probability and Odds

Deviation	Probability	Odds
$\pm$ 0.6745 s	0.5000	1 to 1
$\pm\sigma$	0.6826	2.15 to 1
$\pm 2\sigma$	0.9546	21 to 1
$\pm 3\sigma$	0.9974	369 to 1

3.5.7. Specification of Measurement Data

The results of statistical analysis of multi-sample data are normally expressed as deviations about a mean value; the deviations are expressed as follows :

(*i*) **Probable error.** The result is expressed as $\bar{q} \pm 0.6745\,\sigma$, which implies that 0.50 (50%) of the readings are within this limit and odds are 1 to 1. Obviously there is an even probability that a new reading will lie within with these limits.

(*ii*) **Standard deviation.** The result is expressed as $\bar{q} \pm \sigma$. In this case, the error limit is the standard deviation. It implies that 0.6828 (about 68%) of the readings are within the limits $\sigma = \pm 1$ and the odds are 2.15 to 1. Thus there is approximately a 2 to 1 possibility that a new reading will fall within this limit.

(*iii*) **$\pm$ 2σ limit.** The result is expressed as $\bar{q} \pm 2\sigma$, which implies that 0.9546 (about 95%) of the readings are within the given limits and the odds are 21 to 1.

(*iv*) **$\pm$ 3 σ limit.** The result is expressed as $\bar{q} \pm 3\sigma$, which implies that 0.9974 (about 99%) of the readings are within the given limits. The odds of any observation falling within this limit are 369 to 1.

3.5.8. Confidence Interval and Confidence Level

The range of deviation from the mean value within which a certain fraction of all values are expected to lie is called **confidence interval**.

The probability that the value of a randomly selected observation will lie in the range of confidence interval is termed as **confidence level.**

When the number of observations is large and their errors are random and follow Gaussian distribution, the various confidence intervals about the mean value $\overline{q}$ are given in table 3.3.

TABLE 3.3 Confidence Intervals

Confidence level	*Confidence interval*	*Values lying outside confidence interval*
0.500	$\overline{q} \pm 0.674\ \sigma$	1 in 2
0.800	$\overline{q} \pm 1.282\ \sigma$	1 in 5
0.900	$\overline{q} \pm 1.645\ \sigma$	1 in 10
0.950	$\overline{q} \pm 1.960\ \sigma$	1 in 20
0.990	$\overline{q} \pm 2.576\ \sigma$	1 in 100
0.999	$\overline{q} \pm 3.291\ \sigma$	1 in 1000

- If the number of observations is small (say less than 20) and the standard deviation is not accurately known, the confidence interval must be *broadened.* The standard deviation, in this case is computed as: $s = \sqrt{\dfrac{\Sigma d^2}{n-1}}$; this standard is multiplied by a suitable factor to establish the confidence interval.

3.5.9. Probability Graph Paper

Prior to undertaking statical analysis of the measurement data, the experimenter needs to check that the data is in confirmity with the normal distribution. To accomplish this, the data is plotted on a probability graph paper wherein the abscissa is the value of a particular reading (on a linear scale) and the ordinate has a percentage of readings at or below the value of abscissa (on a logarithmic scale). A straight line is then drawn through the data points; however, while fitting the curve much weightage is not given to the points at low and high ends of the plot. *If the 50 percent reading at the ordinate exactly corresponds to averages of all the data points, the data is considered to follow the Gaussian distribution.*

3.5.10. Rejection of Test Data-Chauvenet's Criterion

During experimentation, it is observed that some points are noticeably different from the majority of the data. Such points can be determined and subsequently eliminated by applying Chauvenet's criterion.

Chauvenet's criterion states as follows:

"*An observed reading may be rejected if the probability of obtaining the particular deviation from the mean is less than* $\dfrac{1}{2n}$". The quantity n refers to the number of observations which are large enough to presume that the data follows the Gaussian error distribution.

Let us consider that during a measurement act ten readings have been made. Then,

$$\frac{1}{2n} = \frac{1}{2 \times 10} = 0.05 \text{ or } 5\%$$

Thus readings with 95 percent of probability are to be accepted, and less than that can be rejected.

Corresponding to probability

$$P(y) = \frac{0.95}{2} = 0.475; y = 1.96$$

Results based on such calculations for different number of readings are given in table 3.4.

TABLE 3.4 Chauvenet's Criterian for Rejecting Data

Number of readings	Ratio of maximum acceptable deviation to standard-deviation, d_{max}/σ
2	1.15
3	1.38
4	1.54
5	1.65
6	1.73
7	1.80
10	1.96
15	2.13
25	2.33
50	2.57
100	2.81
300	3.14
500	3.29
1000	3.48

When applying Chauvenet's criterion, in order to eliminate any dubious data, the following *procedure* is adopted :

(*i*) Using all data points calculate the standard deviation.

(*ii*) Compare the deviation of the individual readings with standard deviation.

(*iii*) If the ratio of deviation of a reading to the standard deviation exceeds the limits given in table 3.4, reject the readings.

(*iv*) Calculate again the mean value and the standard deviations by excluding the rejected readings from the data.

- A criterion (*based upon "confidence intervals"*) used for discarding a data point is that its deviation from the mean exceeds four times the probable error of a single reading. This results in discarding a data outside a confidence interval for a single reading at a confidence level of 0.993.

— A better criterion which does not involve the evaluation of probable error when set of data points is small and standard deviation is not accurately known, *is to discard a reading that lies outside the interval corresponding to confidence level of 0.99 for a single observation* (Refer to tables 3.3 and 3.4). On this basis not more than one reading in 100 would lie outside their range.

— A still better method is to use confidence interval corresponding to a confidence level of 0.95 in order to scrutinize the measurement procedure adopted.

Example 3.19

10 copper rods, selected at random, were found to a have the following lengths in metres :5.30, 5.73, 6.77, 5.26, 4.33, 5.45, 6.09, 5.64, 5.81, 5.75.

Determine any reading that can be rejected by applying Chauvenet's criterion. The ratio of maximum deviation to standard deviation should not exceed 1.96.

Solution. The computations in a tabular form are given below :

S. No.	Length q, m	Deviation, $d = q - \overline{q}$	d^2	$\frac{\lvert d \rvert}{s}$
1	5.30	− 0.313	0.09797	0.5
2	5.73	+ 0.117	0.01369	0.19
3	6.77	+ 1.157	1.33865	1.85
4	5.26	− 0.353	0.12461	0.56
5	4.33	− 1.283	1.64609	2.05
6	5.45	− 0.163	0.02657	0.26
7	6.09	+ 0.477	0.22753	0.76
8	5.64	+ 0.027	0.00073	0.04
9	5.81	+ 0.197	0.03881	0.31
10	5.75	+ 0.137	0.01877	0.22
$n = 10$	$\Sigma q = 56.13$ $\overline{q} = \frac{56.13}{10} = 5.613$		$\Sigma d^2 = 3.53342$	

Arithmetic mean, $\overline{q} = \frac{\Sigma q}{n} = \frac{56.13}{10} = 5.613$

Standard deviation, $s = \sqrt{\frac{\Sigma d^2}{n-1}}$ ... since the number of readings is less than 20.

$$= \sqrt{\frac{3.53342}{10}} = 0.626$$

It is given that for 10 readings the ratio of deviation to standard deviation is not to exceed 1.96 and therefore **reading number 5 *i.e.*, 4.33 m** *should be rejected* (Ans.).

3.5.11. Method of Least Squares

- Let us consider a set of n readings $q_1, q_2, q_3, ... q_n$. The sum of squares of their deviations for some mean value Q_m is,

$$S = \sum_{i=1}^{n} (q_i - Q_m)^2$$

For minimizing S with respect to mean value Q_m, we have:

$$\frac{\partial S}{\partial Q_m} = 2\left(\sum_{i=1}^{n} q_i - nQ_m\right) = 0$$

or,
$$Q_m = \frac{1}{n}\sum_{i=1}^{n} q_i$$

But,
$$\frac{1}{n}\sum_{i=1}^{n} q_i = \overline{q}$$

$\therefore$
$$Q_m = \overline{q} \qquad ....(3.33)$$

Thus the *mean value which minimizes the sum of the squares of the deviation is the arithmetic mean.*

- Now consider two variables u and v measure over a range of values and let us find the linear function connecting y and x.

Let the linear function is :

$$v = au + \mathrm{b}$$

Now let us minimize the quantity,

$$S = \sum_{i=1}^{n} [v_i - (au_i + b)]^2$$

This is accomplished by taking derivatives of S w.r.t. a and b and setting them equal to zero. This gives:

$$nb + a\Sigma u_i = \Sigma v_i \qquad ...(i)$$

and,
$$b\Sigma u_i + a\Sigma u_i^2 = \Sigma u_i\, v_i \qquad ...(ii)$$

Solving the above eqns. (i) and (ii), we get:

$$a = \frac{n\Sigma u_i v_i - (\Sigma u_i)(\Sigma v_i)}{n\Sigma u_i^2 - (\Sigma u_i)^2} \qquad ...(3.34)$$

$$b = \frac{(\Sigma v_i)(\Sigma u_i^2) - (\Sigma u_i\, v_i)(\Sigma u_i)}{n\Sigma u_i^2 - (\Sigma u_i)^2} \qquad ...(3.35)$$

The standard deviations may be found as :

$$s_a = \sqrt{\frac{n}{n\Sigma u_i^2 - (\Sigma u_i)^2}}\, s_v \qquad ...(3.36)$$

and,
$$s_b = \sqrt{\frac{\Sigma u_i^2}{n\Sigma u_i^2 - (\Sigma u_i)^2}}\, s_v \qquad ...(3.37)$$

where, $s_v = \sqrt{\frac{1}{n}\sum (au_i + b - v_i)^2}$ ...(3.38)

Standard deviation of u is,

$$s_u = \sqrt{\frac{1}{n}\sum\left(\frac{v_i - b}{a} - u_i\right)^2} = \frac{s_v}{a} \qquad ...(3.39)$$

Example 3.20

The data given below are expected to follow a linear relationship:

$$v = au + b$$

u	1.8	4.6	6.6	9.0	11.4	13.4
v	2.2	3.2	5.2	6.4	8.0	10.0

(i) *Obtain the best linear relation, and*

(ii) *Calculate the standard deviation.*

Solution. *Given :* $v = au + b$ .... Linear relationship:

Number of reading $n = 6$,

The computations in a tabular from are given below:

u_i	v_i	$u_i v_i$	u_i^2
1.8	2.2	3.96	3.24
4.6	3.2	14.72	21.16
6.6	5.2	34.32	43.56
9.0	6.4	57.60	81.0
11.4	8.0	91.20	129.96
13.4	10.0	134.00	179.56
$\Sigma u_i = 46.8$	$\Sigma v_i = 35$	$\Sigma u_i v_i = 335.8$	$\Sigma u_i^2 = 458.48$

(*i*) **Best linear relation :** Using equations (3.34) and (3.35), we have:

$$a = \frac{(6)\times(335.8)-(46.8)\times(35)}{(6)\times(458.48)-(46.8)^2} = \frac{376.8}{560.64} = 0.672$$

$$b = \frac{(35)\times(458.48)-(335.8)\,(46.8)}{(6)\,(458.48)-(46.8)^2} = \frac{331.36}{560.64} = 0.591$$

The best linear relation is : $v = 0.672u + 0.591$ **(Ans.)**

(*ii*) **Standard deviation:** The computations for finding standard deviation, in a tabular form, are given below:

u_i	v_i	$au_i + b$	$au_i + b - v_i$	$(au_i + b - v_i)^2$
1.8	2.2	1.8006	− 0.3994	0.1595
4.6	3.2	3.6822	+ 0.4822	0.2325
6.6	5.2	5.0262	− 0.1738	0.0302
9.0	6.4	6.6390	+ 0.239	0.0571
11.4	8.0	8.2518	+ 0.2518	0.0634
13.4	10.0	9.5958	− 0.4074	0.1660
				$\Sigma(au_i + b - v_i)^2 = 0.7087$

∴ Standard deviation of v,

$$s_v = \sqrt{\frac{1}{n}\Sigma(au_i + b - v_i)^2} = \sqrt{\frac{0.7087}{6}} = \pm\mathbf{0.3436}\ \textbf{(Ans.)}$$

From eqn. (3.39), standard deviation of u,

$$s_u = \frac{s_v}{a} = \frac{\pm 0.3436}{0.672} = \pm\mathbf{0.511}\ \textbf{(Ans.)}$$

From eqns. (3.36) and (3.37) the standard deviations in a and b are ;

$$s_a = \sqrt{\frac{n}{n\Sigma u_i^2 - (\Sigma u_i)^2}}\ s_v$$

$$= \sqrt{\frac{6}{6\times 458.48 - (46.8)^2}} \times (\pm 0.3436) = \pm\mathbf{0.0355}\ \textbf{(Ans.)}$$

$$s_b = \sqrt{\frac{\Sigma u_i^2}{n\Sigma(u_i^2) - (\Sigma u_i)^2}}\ s_v$$

$$= \sqrt{\frac{458.48}{6\times 458.48 - (46.8)^2}} \times (\pm 0.3436) = \pm\mathbf{0.3107}\ \textbf{(Ans.)}$$

Example 3.21

(a) Enumerate the factors which should be considered while selecting an instrument for a specific measurement application.

(b) The iron losses in a ferromagnetic material used in a transformer vary with frequency f of the supply given to the transformer. For a particular transformer these iron losses were determined at various frequencies with a constant flux density in the ferromagnetic material; the flux results obtained are given below :

Frequency, f (Hz) :	*550*	*700*	*850*	*1000*
Iron losses, P(mW) :	*23*	*31*	*47*	*61*

Assuming the iron losses to have general form $P = af^2 + bf$, determine the constant a and b to achieve best fit for the four measured values, using method of least squares.

Solution. (*a*) The following *factors* need to be considered for the selection of an instrument to be used for a specific measurement application :

1. Easy calibration.
2. Convenience and ease in reading the instrument.
3. The maximum and minimum values the input variables are expected to assume.
4. Nature of the input signal.
5. Non-interference with measuring system.
6. Accuracy expected from the instrument.
7. Cost criterion.
8. Specific application of the data.

9. Required form of data.

10. Negligible or well-known environmental errors.

(*b*) *Given :* General form $P = af^2 + bf$

or,
$$\frac{P}{f} = af + b$$

The above relation may be expressed as:

$$v = au + b$$

which is a straight line relationship between, $v = \frac{P}{f}$ and $u = f$,

i.e.,	$u = f$:	550	700	850	1000
	$v = \frac{P}{f}$:	0.04182	0.04429	0.05529	0.061

The computations are tabulated below:

u_i	v_i	$u_i v_i$	u_i^2
550	0.04182	23	0.302×10^6
700	0.04429	31	0.49×10^6
850	0.05529	47	0.722×10^6
1000	0.0610	61	1×10^6
$\Sigma u_i = 3100$	$\Sigma v_i = 0.2024$	$\Sigma u_i v_i = 162$	$\Sigma u_i^2 = 2.514 \times 10^6$

In order to find the values of *a* and *b* for the best straight line fit using the method of least squares, let us use the equations (3.34) and (3.35) as follows:

$$a = \frac{n\Sigma u_i v_i - (\Sigma u_i)(\Sigma v_i)}{n\Sigma u_i^2 - (\Sigma u_i)^2} \qquad \text{...[Eqn (3.34)]}$$

$$= \frac{4 \times 162 - (3100)\,(0.2024)}{4 \times 2.514 \times 10^6 - (3100)^2} = \frac{20.56}{446000} = 46.1 \times 10^{-6}$$

$$b = \frac{(\Sigma v_i)\,(\Sigma u_i^2) - (\Sigma u_i v_i)\,(\Sigma u_i)}{n\Sigma u_i^2 - (\Sigma u_i)^2}$$

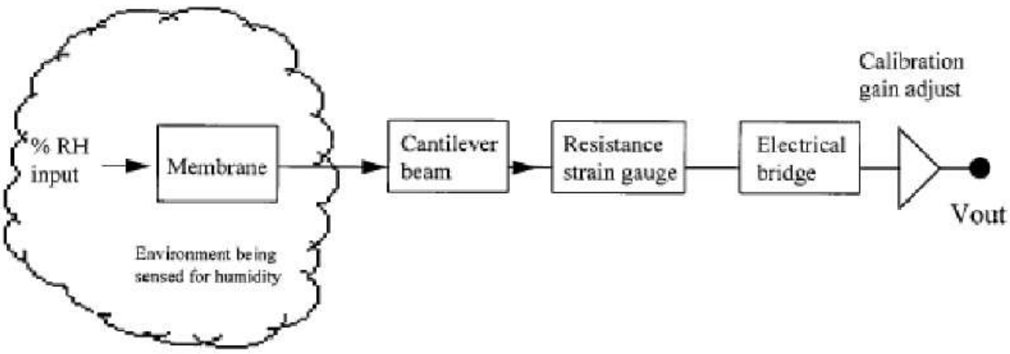
$$= \frac{0.2024 \times 2.514 \times 10^6 - 162 \times 3100}{4 \times 2.514 \times 10^6 - (3100)^2}$$

$$= \frac{6633.6}{446000} = 14.87 \times 10^{-3}$$

Hence, *the best fit relationship is:*

$$P = 46.1 \times 10^{-6} f^2 + 14.87 \times 10^{-3} f \text{ mW } \textbf{(Ans.)}$$

STATIC PERFORMANCE OF INSTRUMENT-



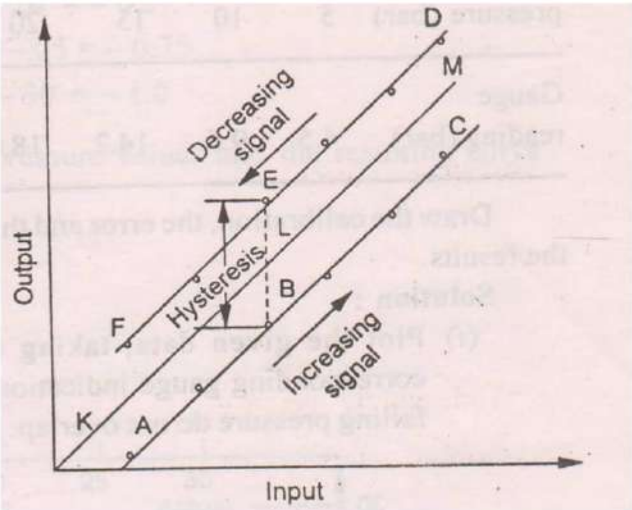
The **static characteristics** of instruments are related with steady state response.The relationship between the output and the input when the input does not change, or the input is changing at a slow rate.

CALIBRATION-Parameters

- **Range & Span**
- **Accuracy & Precision**
- **Threshold & Resolution**
- **Sensitivity**
- **Linearity**
- **Drift**
- **Hysteresis & Dead Zone**
- **Reproducibility**
- **Uncertainty**
- **Traceability**
- **Readability**

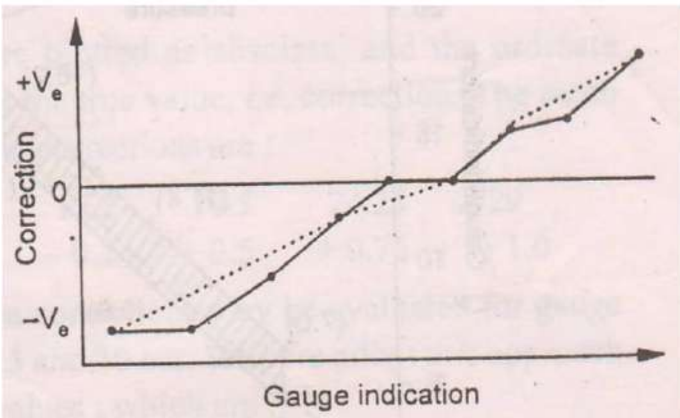
Calibration:

Calibration consists of comparing the output of the instrument or sensor (under test) against the output of an instrument of known accuracy (higher accuracy) when the same input (the measured quantity is applied to both instrument)



Calibration Curve

The procedure is carried out for a range of inputs covering the whole measurement range of the instrument or sensor Ensures that the measuring accuracy of all instruments and sensors used in a measurement system is known over the whole measurement range, provided that the calibrated instruments and sensors are used in environmental conditions that are the same as those under which they were calibrated as shown in figure**.**



Correction curve

Range: The region between the limits within which an instrument is designed to operate for measuring.

For a standard thermometer this is 0 to 100°C. This is the same as the full scale.

Span: The algebraic difference between higher calibration values to lower calibration values.

For above e.g. span is 100-0 = 100°C.

Accuracy - is how close the measurement to the true value is. It is more easily quantified by percentage error where:

Percentage error = (indicated value – true value) / true value x 100%

It can also be related to the percentage error of the full measuring range:

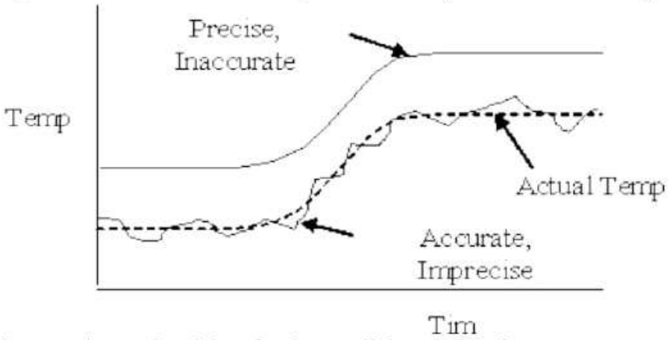
= (indicated value – true value) / Maximum scale value x 100%

Therefore, if a thermometer is defined as having an accuracy of +-2.5% with a range of 0 to 50°C, then the maximum error is 1.25°C. This error may not be a problem when measuring in the 40 to 50°C range but at the lower end, 0 to 10°C, an error of +-1.25°C is quite large if defined using the true value. It is important to select an instrument so that it is not normally operating at the extremes of its measuring range.

Precision – is the capability of an instrument to reproduce a reading.

$$\textbf{Precision} = (X_{maximum} - X_{average}) \times 100 / X_{average}$$

Precision may be defined as the reproducibility with which repeated measurements of the same



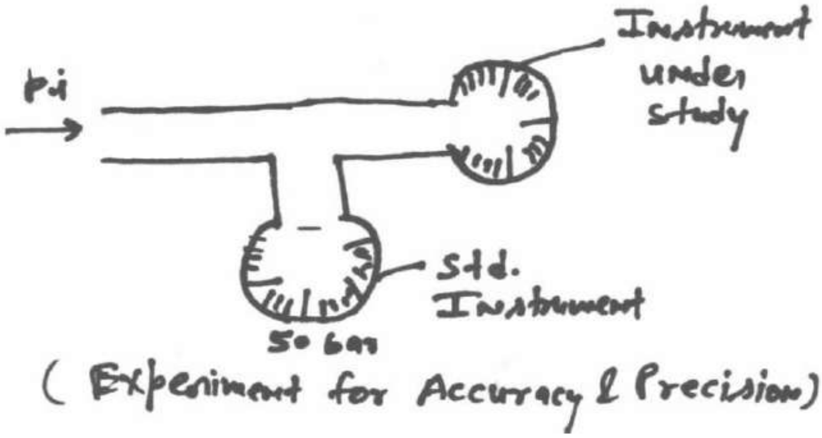
variable can be made under identical conditions. An instrument can be precise but inaccurate and, likewise, it is possible to have an accurate but imprecise instrument. See below:

In this figure pressure is applied at inlet & we measure the pressure at outlet. There are two instruments which records the pressure.

1. Standard Instrument (at 50 bar)
2. Instrument under study (shows various reading)

Let us make a graph between input & output signal & we find out as shown

For understanding the accuracy & precision we conduct an experiment on a pressure gauge as shown in figure



Calibration values of a pressure gauge.

From reading accuracy is –

Accuracy = (indicated $value_{max}$ – true value) / true value x 100%

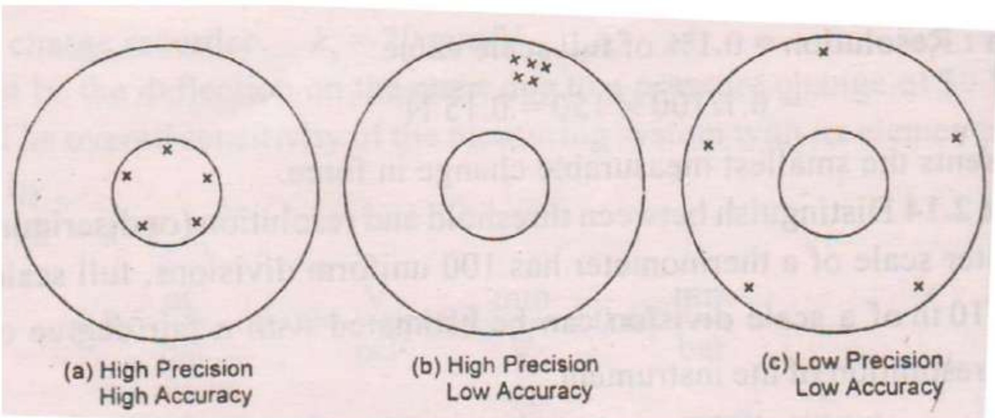
i.e (53-50) /50 x 100 % = +6%

From reading Precision is –

Precision = ($X_{maximum} - X_{average}$) x 100 / $X_{average}$

i.e 53-52/52 x 100 % = +2%

Example: (game of darts)



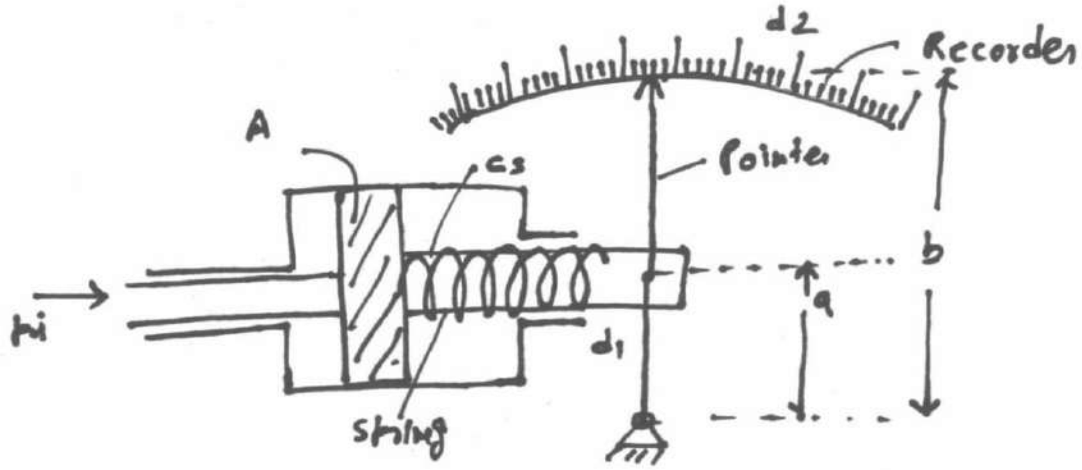
so from data accuracy i.e inaccuracy in a system is always more than precision(i.e imprecision).

Because the factor which are responsible for imprecision are lies within the instrument & factor which are lies inside & outside the instrument (like atmospheric conditions , nearby power lines , Mounting Conditions etc) . So factor responsible are more in case of inaccuracy.

Threshold & Resolution :

Threshold maybe defined as the minimum change in input signal to make the pointer move from zero value.

For understanding let us take the e.g of piston type pressure gauge as shown in figure



(Piston cylinder Pressure gauge)

where p_i = Input pressure ; $\frac{b}{a}$, leverage

$$\boxed{d_2 = \frac{A}{C_s}\left(\frac{b}{a}\right) p_i}$$ – signal Relation

From figure as the pressure is increase (i.e Δp_i) from any zero value the pointer is likely starts to move with displacement d2.

Hence Δp_i is known as the threshold in system.

Resolution: Resolution maybe defined as the minimum change in input signal to make the pointer move from any Non- zero value.

Note: (Threshold value at non-zero point is called resolution & Resolution at zero point is called threshold.)

Sensitivity: Sensitivity of an instrumentation system is the ratio of magnitude of response (i.e. incremental change in output to input signal)

Sensitivity = Δ Xo/ Δ Xi

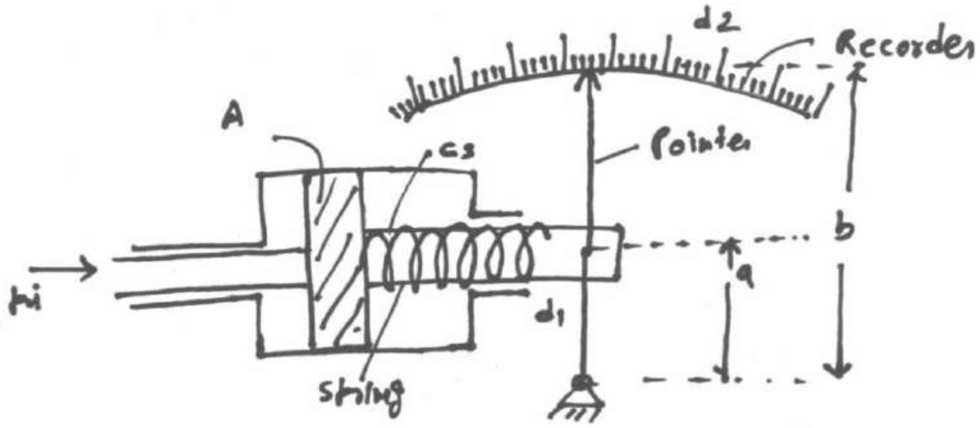
Where Δ Xo - incremental change in output
Δ Xi - incremental change in input

Note sensitivity of an instrument not a dimensionless quantity. For
Units = $\Omega/^{o}C$ (in case of Resistance Thermometer)
Units = mm/bar (In case of Pressure gauge piston type)

Overall sensitivity – a measurement system consists of a number of devices. If the sensitivities of these devices are K1, K2, K3, etc., then the overall sensitivity is the product of the individual sensitivities, K1xK2xK3xetc

For a pressure gauge as shown the sensitivity relation is given

From above figure :It is clear that sensitivity depends upon so many factors i.e



(Piston cylinder Pressure gauge)

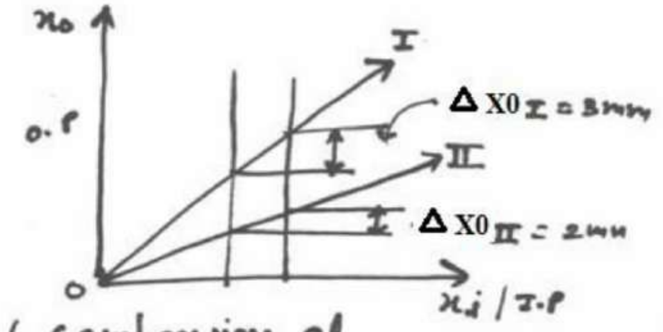
where p_i = Input pressure ; $\frac{b}{a}$, leverage

$$\boxed{d_2 = \frac{A}{C_s}\left(\frac{b}{a}\right) p_i}$$ – Signal Relation

1. Area of piston(A) 2. Spring Constant (Cs) 3. Leverage(b/a)

Comparison of Sensitivity of two instruments:

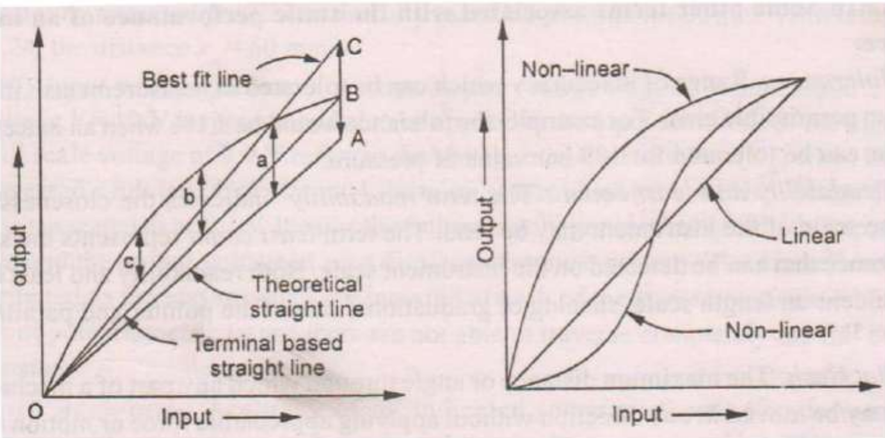
Let us take two instruments having sensitivity relation S1 & s2 respectively & at the same input the output change in instrument 1 is more as compared to instrument 2 as shown by graph.



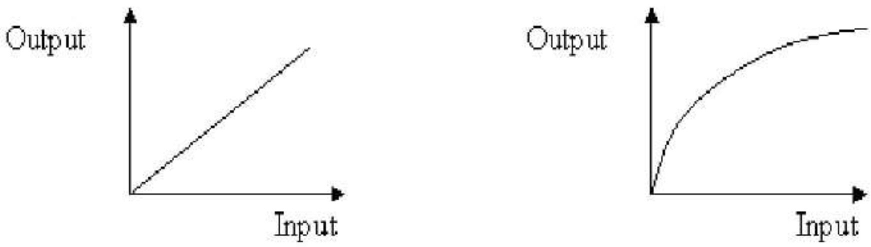
(comparision of Sensitivity of two Instruments) where $\Delta x_{i_I} > \Delta x_{i_{II}}$ eg Pressure gauge.

From figure it is clear that the instrument having more sensitivity shows the larger value of tangent (tanø).

Linearity: This is usually referred to as non-linearity. It is the difference between actual and ideal straight line behavior. One way to define non linearity is to divide the maximum non linearity value by the full scale deflection.



Linear **Nonlinear**



The linearity of an instrument may be defined as deviation of the reading from the linear characteristics.

Reasons of Non-Linerity :

- Non –Linear elements of instruments
- Mechanical Hysteresis
- Viscous flow or creep

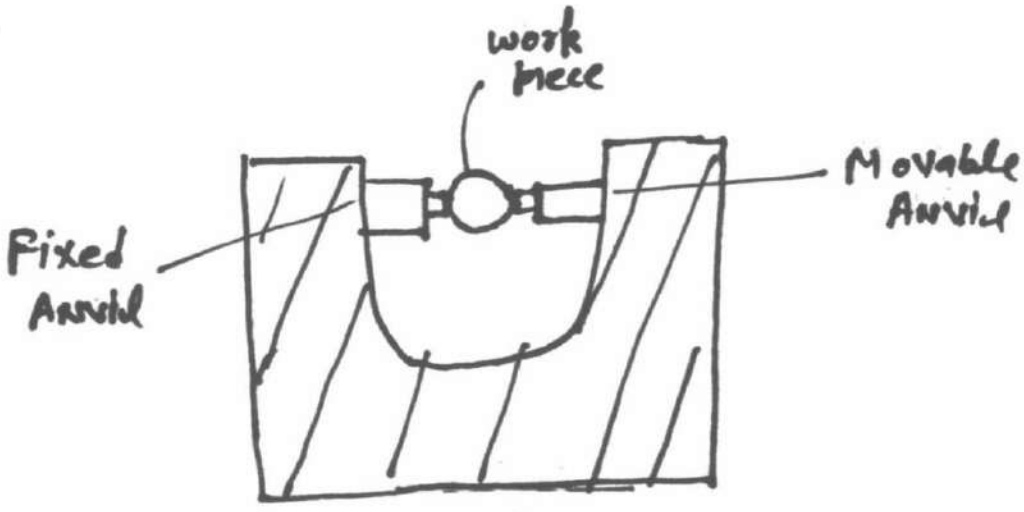
Drift: a measurement must be made with respect to a known datum or base line. It is very common and convenient to adjust the output of the instrument to zero at the datum. For example, a thermometer is set up to display zero at the freezing point of water. A pressure gauge is adjusted to read zero when open to atmosphere.

A common problem with instruments is that the output at the datum drifts and introduces an error to the measurement. All sensors are affected by drift to some extent whether it be short term or long term. Short term drift is usually associated with changes in temperature or electronics stablising. Long term drift is usually associated with aging of the transducer.

Types of Drift:

- Zero Drift
- Sensitivity drift

Zero Drift-defind as the drift from zero value during usage of an instrument Example screw gauge as shown in figure:



(screw gauge)
e.g of zero Drift

Sensitivity drift: occurs due to some atmospheric change is called sensitivity drift. E.g. pressure gauge:

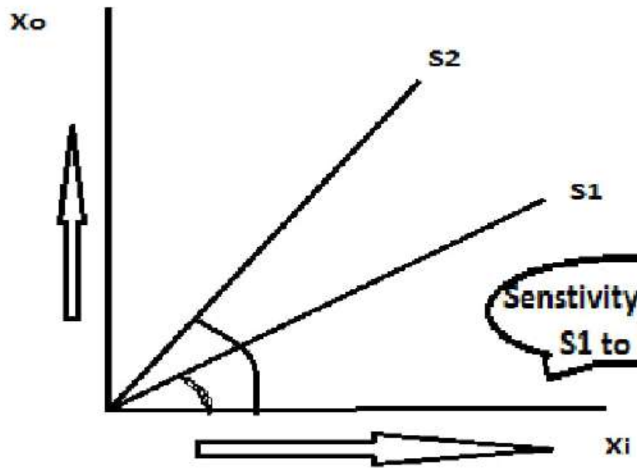
In Pressure gauge sensitivity drift calculated as :

$$\text{Sensitivity Drift} = \Delta Xo/\Delta Xi = (A/Cs) \text{ X } b/a$$

As temperature is increased the

- Cs reduces (i.e. spring becomes softer) so deflection is more.
- Friction b/w piston increase.

Hence sensitivity is shifted as shown in figure by S1 to S2

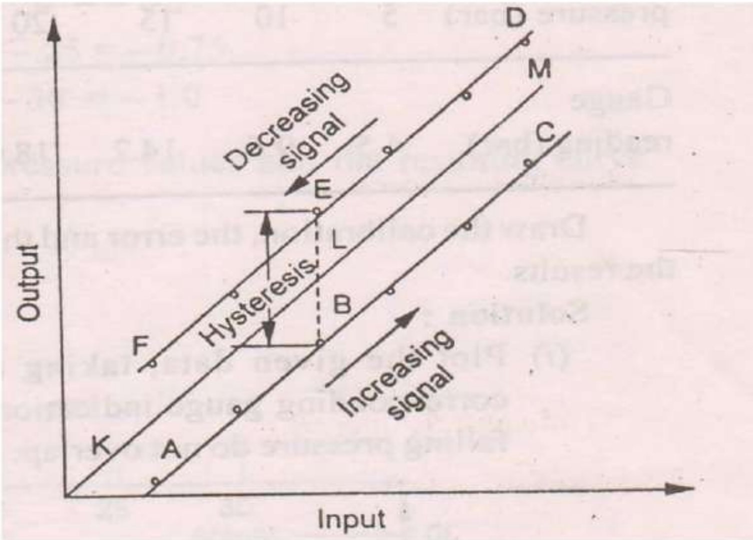


- Sensitivity drift effect the error.

Hysteresis.:

Hysteresis is the maximum difference between the same measured quantity (input signal) between the upscale & downscale readings during a full range transverse in each direction.

This refers to the situation where different readings (outputs) are sometimes observed for the same input because the input was approached from different directions. For example a thermometer exposed to an increasing temperature input (i.e. going from 0 to 100°C) may show a slightly different profile to that for the decreasing input (i.e. decreasing from 100 to 0 °C).



Hysteresis Loss

Hysteresis is often noticeable in mechanical systems where degradation of parts due to wear create slightly different results when the direction of the input is reversed. Imagine two cogs that have small gaps between the teeth due to wear. This will create hysteresis.

Dead Zone: is the largest zone through which an input signal can be varied without initiating from the indicating instrument.

- Friction & play is the direct cause of Dead band & Zone.

Backlash: The maximum distance through which one part of the instrument is moved without disturbing the other part.

Reproducibility: It is the consistency of pattern of variation in measurement. When individual measurements are carried out the closeness of the agreement between the results of measurements of the same quantity.

Uncertainty: The range about the measured value within the true value of the measured quantity is likely to lie at the stated level of confidence.

Traceability: It is nothing establishing a calibration by step by step comparison with better standards.

THE DYNAMIC CHARACTERISTICS OF ANY MEASUREMENT SYSTEM ARE:

(i) Speed of response and Response time
(ii) Lag
(iii) Fidelity
(iv) Dynamic error
(v) Overshoot

Out of the above four characteristics the Speed of Response and the Fidelity are desirable in a dynamic system, while Lag and Dynamic error are undesirable.

(i) Speed of Response and Response Time

Speed of Response is defined as the rapidity with which an instrument or measurement system responds to changes in measured quantity.

Response Time is the time required by instrument or system to settle to its final steady position after the application of the input. For a step input function, the response time may be defined as the time taken by the instrument to settle to a specified percentage of the quantity being measured, after the application of the input. This percentage may be 90 to 99 percent depending upon the instrument. For portable instruments it is the time taken by the pointer to come to rest within 0.3 percent of final scale length and for switch board (panel) type of instruments it is the time taken by the pointer to come to rest within 1 percent of its final scale length.

(ii) Measuring Lag

As discussed earlier, an instrument does not react to a change in input immediately. The delay in the response of an instrument to a change in the measured quantity is known as *measuring lag*. Thus it is the retardation delay in the response of a measurement system to changes in the measured quantity. This lag is usually quite small, but this small lag becomes highly important when high speed measurements are required. In the high speed measurement systems, as in dynamic measurements, it becomes essential that the time lag be reduced to minimum.

Measuring lag is of two types

i) **Retardation type:** In this type of measuring lag the response begins immediately after a change in measured quantity has occurred.
ii) **Time delay:** In this type of measuring lag the response of the measurement system begins after a dead zone after the application of the input.

(iii) Fidelity

Fidelity of a system is defined as the ability of the system to reproduce the output in the same form as the input. It is the degree to which a measurement system indicates changes in the measured quantity without any dynamic error. Supposing if a linearly varying quantity is applied to a system and if the output is also a linearly varying quantity the system is said to have 100 percent fidelity. Ideally a system should have 100 percent fidelity and the output should appear in the same form as that of input and there is no distortion produced in the signal by the system. In the definition of fidelity any time lag or phase difference between output and input is not included.

(iv) Dynamic Error

The dynamic error is the difference between the true value of the quantity changing with time and the value indicated by the instrument if no static error is assumed.

However, the total dynamic error of the instrument is the combination of its fidelity and the time lag or phase difference between input and output of the system.

(v) **Overshoot**

Moving parts of instruments have mass and thus possess inertia. When an input is applied to instruments, the pointer does not immediately come to rest at its steady state (or final deflected) position but goes beyond it or in other words overshoots its steady position.

The overshoot is evaluated as the maximum amount by which moving system moves beyond the steady state position. In many instruments, especially galvanometers it is desirable to have a little overshoot but an excessive overshoot is undesirable.

Overshoot response graph

A typical overshoot response graph can be shown as the response time stated in terms of rise time, peak percentage overshoot and settling time. Such an under damped graph in control system technology of a measuring instrument is shown in Fig. .

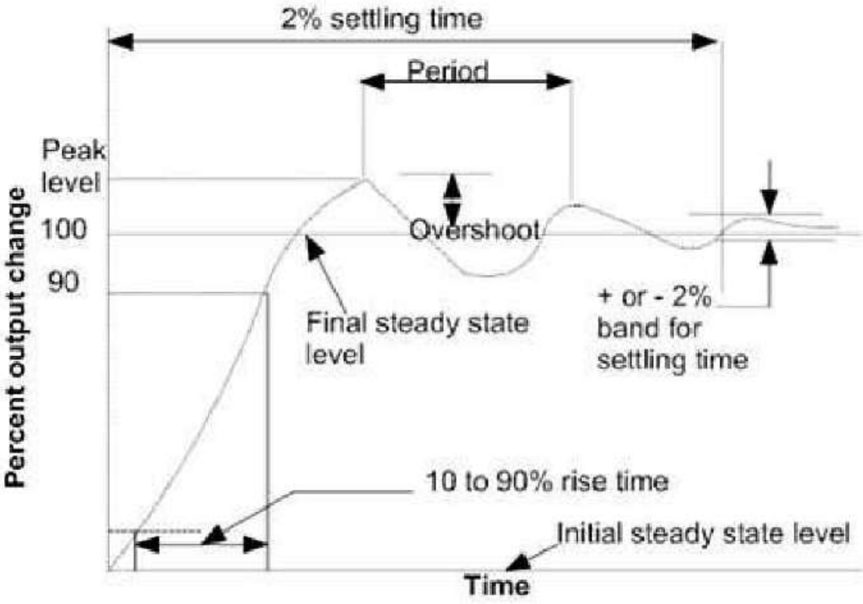


Fig. Overshoot response graph

2.4.2. Dynamic Characteristics of a Measurement System

The *dynamic characteristics* of a measurement system are:

1. Speed of response ... *Desirable*
2. Measuring lag ... *Undesirable*
3. Fidelity ... *Desirable*
4. Dynamic error ... *Undesirable*

1. **Speed of response.** The *"speed of response"* or *"responsiveness"* is defined as the *rapidity with which a measurement system responds to changes in the measured quantity.*
2. **Measuring lag.** It refers to *retardation or delay in the response of a measurement system to changes in measured quantity.*

   The lag *is caused by conditions such as capacitance, inertia, or resistance.*

   The measuring lags are of the following two types:

   (*i*) ***Retardation type lag.*** In this type of measuring lag the response of the measurement system *begins immediately after a change in measured quantity has occurred.*

   (*ii*) ***Time delay type lag.*** In this case the response of the measurement system *begins after a dead time after the application of the input.* If the measured quantity varies at a fast rate, the dead time has a severe adverse effect on the performance of the system. The measurement lags of this type are *very small* and are of the order of a fraction of a second and hence can be *ignored.*
3. **Fidelity.** It is defined as the *degree to which a measurement system indicates changes in the measured quantity without any dynamic error.* It refers to the ability of the system to reproduce the output in the same form as the input.
4. **Dynamic error.** The dynamic error, also called **"measurement error"**, is the *difference between the true value of the quantity changing with time and the value indicated by the measurement system if no static error is assumed* (see Fig. 2.5.)

   - *The maximum amount by which the pointer moves beyond the steady state is called* ***overshoot.***

Example 2.16

On the application of a step input of 80 bar to a pressure gauge, the pointer swings to pressure of 82.5 bar and finally comes to rest at 81.3 bar. Determine:

(*i*) *The overshoot of the gauge reading and express it as a percentage of the final reading.*

(*ii*) *The percentage error of the gauge.*

Solution. (*i*) *Overshoot of the gauge reading* = 82.5 – 81.3 = **1.2 bar** **(Ans.)**

$$\text{Percentage overshoot} = \frac{1.2}{81.3} \times 100 = \mathbf{1.476\%} \quad \textbf{(Ans.)}$$

(*ii*)
$$\text{Percentage error} = \frac{81.3 - 80}{80} \times 100 = \mathbf{1.625} \quad \textbf{(Ans.)}$$

2.4.3. 'Dynamic' Analysis of Measurement Systems

The dynamic behaviour of measurement systems is studied in the following two domains:

1. Time domain analysis.
2. Frequency domain analysis.

1. Time domain analysis:

In this the input signal is applied to the measurement system and the behaviour of the system is studied as a *function of time.* The dynamic response of the system to different types of inputs, which are a function of time is analysed at different intervals of time after the application of the input signal. In most cases, the actual input signals vary in random fashion with respect to time and therefore cannot be mathematically defined. Consequently the performance of a system can be analysed (in the time domain analysis) by using the following standard *test signals/inputs*:

(*i*) Step input;

(*ii*) Ramp input;

(*iii*) Parabolic input;

(*iv*) Impulse input.

2. Frequency domain analysis:

This type of analysis of a system *pertains to the steady state response of the system to a sinusoidal input.* Here, the system is subjected to a sinusoidal input and the system response is studied with *frequency as the independent variable.*

- *Frequency response.* It is the maximum frequency of the measured variable that an instrument is capable of following without error. The usual requirement is that the frequency of measurand should not exceed 60 per cent of the natural frequency of the measuring instrument.

Standard test signals/inputs:

The most common standard inputs used for dynamic analysis are discussed below:

1. Step function:

Refer to Fig. 2.9(*a*). It is a *sudden change from one steady value to another.*

It is mathematically represented by the relationship:

$$x = 0 \quad \text{at} \quad t < 0$$
$$x = x_c \quad \text{at} \quad t \geq 0$$

where x_c is a constant value of the input signal x_i.

- The *"transient response" indicates the capacity of the system to cope with changes in the input signal.*

2. Ramp or linear function:

In this case, the *input varies linearly with time.*

This input is mathematically represented as:

$$x = 0 \quad \text{at} \quad t < 0$$
$$x = \psi \quad \text{at} \quad t \geq 0$$

where, ψ is the slope of the input versus time relationship.

- *The ramp-response becomes indicative of the steady state error in following the changes in the input signal.*

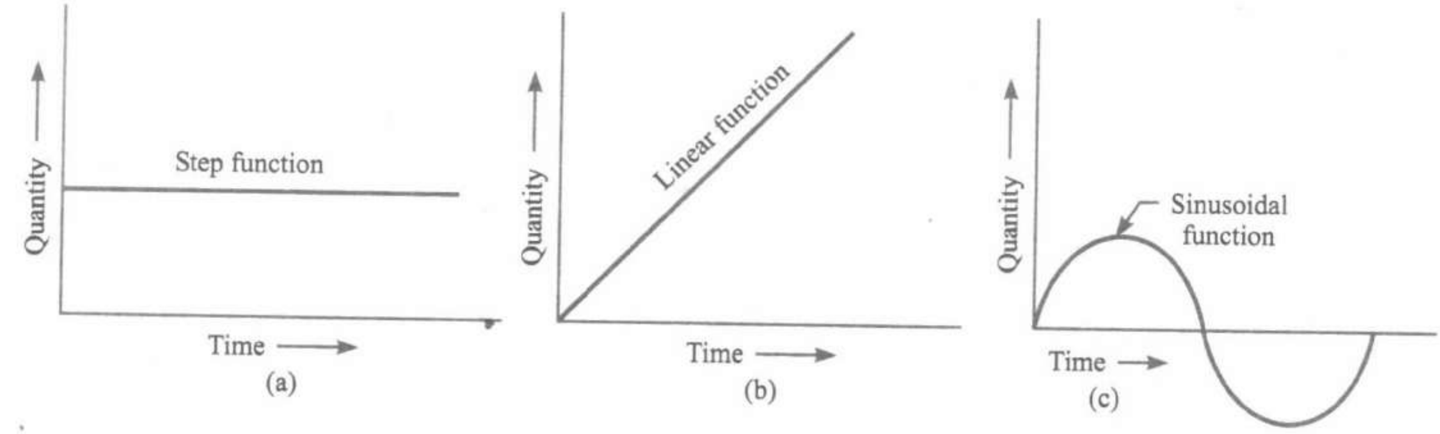


Fig. 2.9. Standard input functions.

3. Sinusoidal function:

In this case the *input varies sinusoidally with a constant maximum amplitude.*

It is represented mathematically as follows:

$$I_i = A \sin \omega t$$

where, A = Amplitude, and

ω = Frequency in rad/s.

- *The frequency or harmonic response is a measure of the capability of the system to respond to inputs of cyclic nature.*

A **general measurement system** can be mathematically described by the following **differential equation**:

$$(A_n D^n + A_{n-1} D^{n-1} + ... + A_1 D + A_0) I_o = (B_m D^m + B_{m-1} D^{m-1} + ... + B_1 D + B_0) I_i \quad ...(2.11)$$

where, A's and B's = Constants, depending upon the physical parameters of the system,

D^k = Operative derivative of the order k,

I_o = The information out of the measurement system, and

I_i = The input information.

The order of the measurement system is generally classified by the value of the power of n.

- *Zero-order system:* $n = 0$ and $A_1, A_2, A_3 ..... A_n = 0$
- *First-order system:* $n = 1$ and $A_2, A_3, A_4 ..... A_n = 0$
- *Second-order system:* $n = 2$ and $A_3, A_4, A_5 ..... A_n = 0$

The above method of classification is used for most of the instruments and systems.

Although general equation can be solved by various methods, we shall be using method of D-operator for getting its solution.

2.4.4. Zero, First and Second Order Systems

2.4.4.1. Zero-order systems

Fig. 2.10 shows the block diagram of a 'Zero-order system'. In this case the output of the measuring system (ideal) is directly proportional to input, no matter how the input varies. The *output is faithful reproduction of input without any distortion or time lag.*

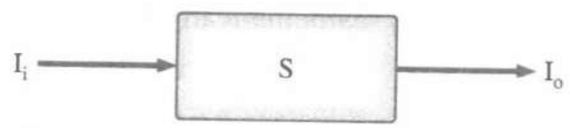


Fig. 2.10. Block diagram for Zero-order system.

The behaviour of the zero-order system is represented by the following mathematical solution:

$$I_o = S\, I_i \qquad ...(2.12)$$

where, I_o = Information out of the measuring system,

S = Sensitivity of the system, and

I_i = Input information

This equation is obtained by putting $n = 0$ in the general equation (2.11),

i.e.,
$$A_o I_o = B_o I_i$$

or,
$$I_o = \frac{B_o}{A_o} I_i = SI_i \qquad ...(2.13)$$

The zero-order system is characterised only by the static sensitivity (parameter), the value of which is obtained through the process of static calibration.

***Examples of zero-order system*:**

- Mechanical levers;
- Amplifiers;
- Potentiometer (It gives an output voltage which is proportional to wiper's displacement) etc.

2.4.4.2. First-order systems

Fig. 2.11 shows the block diagram of a 'First-order system'.

The behaviour of a first-order system is given by following first-order differential equation:

$$A_1 \frac{dI_o}{dt} + A_o I_o = B_o I_i \qquad ...(2.14)$$

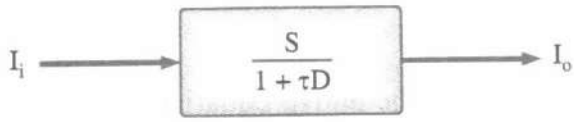


Fig. 2.11. Block diagram for First-order system.

(This equation is obtained by inserting $n = 1$ in the general equation).

Eqn. (2.14) may be written in standard form as follows:

$$\frac{A_1}{A_o}\frac{dI_o}{dt} + I_o = \frac{B_o}{A_o} I_i \qquad ...(2.15)$$

or,
$$\tau \frac{dI_o}{dt} + I_o = S\, I_i \qquad ...(2.16)$$

where, $\tau = \frac{A_1}{A_0}$ = Time constant, and

$S = \frac{B_o}{A_o}$ = Sensitivity.

Using D-operator, we get:

$$\left[\text{where, } D = \frac{d}{dt}, \text{ and } D^2 = \frac{d^2}{dt^2}\right]$$

$$\tau D I_o + I_o = SI_i$$

or,
$$I_o(\tau D + 1) = SI_i$$

or, $$\frac{I_o}{I_i} = \frac{S}{1 + \tau D} \qquad \text{...(2.17)}$$

Equation (2.17) gives the standard form of *transfer operator* for first-order system.

Examples of first-order system :

- Vetocity of a true falling mass;
- Air pressure build-up in bellows;
- Measurement of temperature by mercury-in-glass thermometers;
- Thermistors and thermocouples;
- Resistance-capacitance network.

2.4.4.3. Second-order systems

Fig. 2.12 shows the block diagram of 'Second-order system':

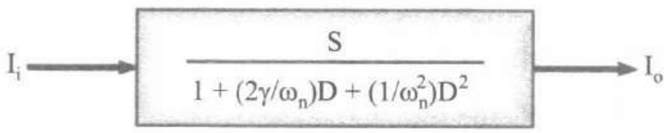


Fig. 2.12. Block diagram for Second-order system.

The behaviour of a second-order system is given by the following differential equation (obtained by putting $n = 2$ in the general equation);

$$A_2 \frac{d^2 I_o}{dt^2} + A_1 \frac{dI_o}{dt} + A_o I_o = B_o I_i \qquad \text{...(2.18)}$$

Dividing the above equation by A_o, we have:

$$\frac{A_2}{A_o} \frac{d^2 I_o}{dt^2} + \frac{A_1}{A_0} \frac{dI_o}{dt} + I_o = \frac{B_o}{A_o} I_i \qquad \text{...(2.19 a)}$$

Let, $$\omega_n = \sqrt{\frac{A_o}{A_2}} = \text{Undamped natural frequency, rad/s,}$$

$$\gamma = \frac{A_1}{2\sqrt{A_0 \ A_2}} = \text{Damping ratio, dimensionless, and}$$

$$S = \frac{B_o}{A_o} = \text{Static sensitivity or steady-state gain.}$$

Then, by substituting these values in eqn. 2.19 (*a*), we get:

$$\frac{1}{\omega_n^2} \cdot \frac{d^2 I_o}{dt^2} + \frac{2\gamma}{\omega_n} \cdot \frac{dI_o}{dt} + I_o = SI_i \qquad \text{...(2.20)}$$

or, in terms of D-operator, we have:

$$\left(\frac{D^2}{\omega_n^2} + \frac{2\gamma}{\omega_n} D + 1 \right) I_o = SI_i$$

or, $$\frac{I_o}{I_i} = \frac{S}{\frac{1}{\omega_n^2} D^2 + \frac{2\gamma}{\omega_n} D + 1} \qquad \text{...(2.21)}$$

- Piezoelectric pick-up;
- Spring-mass system (used for acceleration and force measurements);
- Pen control system on X-Y plotters;
- U.V. galvanometer, etc.

Damping ratio :

In the design of instruments a term which is very frequently used is the *"damping ratio"* (γ) defined as the *ratio of the actual value of coefficient of viscous friction in movement and the value required to produce critical damping.*

i.e.,
$$\gamma = \frac{A_1}{2\sqrt{A_o A_2}}$$

This dimensionless term is very useful because to determine its value, it is not necessary that the values of A_1, A_0 and A_2 may be known. In practice it is not easy to determine accurately the values of A_1 and A_2. Further, even if these values are known, they do not in themselves specify whether the instrument is under, over or critically damped, since a numerical calculation has to be performed with them first. Therefore, designers find "*damping ratio*" as a very convenient measure of the amount of damping present in the movement.

The terms *damping ratio* (γ) and *underdamped natural frequency* (ω_n) immediately conjure up a physical picture of the response of an instrument and both of the quantities are very easy to measure. Thus γ and ω_n easily do away with quantities A_2, A_1 and A_0.

2.4.5. First-order System Responses

The complete solution of an equation which describes the dynamical behaviour of a system consists of the following *two parts*:

(*i*) **Complementary function.** *It corresponds to the short time or transient response.*

(*ii*) **Particular integral.** *It refers to the long time steady state response.*

The transfer operator form of the first-order system is given by :

$$\frac{I_o}{I_i} = \frac{S}{1 + \tau D}$$

When S (static sensitivity or steady state gain) equals *unity*, we get:

$$(1 + \tau D)\, I_o = I_i \qquad \text{...(2.22)}$$

Now we shall obtain the solution of this equation for different standard inputs (The solutions are not mathematically rigorous, but are practical).

Transient response (Complementary function):

The transient response from the auxiliary equation is obtained by putting input I_i equal to zero;

i.e.,
$$(1 + \tau D) I_{o,t} = 0 \qquad \text{...(2.23)}$$

(subscript t refers to the transient value)

Let the solution be of the form :

$$I_{o,t} = A\, e^{mt}$$

where, m is an algebraic variable)

or, $$(1 + \tau D)\, A\, e^{mt} = 0$$

or, $$A\, e^{mt} + \tau \,.\, \frac{d}{dt}(A\, e^{mt}) = 0$$

or, $$A\, e^{mt} + \tau \cdot A\, m\, e^{mt} = 0$$

$$A\, e^{mt}(1 + \tau \cdot m) = 0$$

$\therefore$ $$m = -\frac{1}{\tau}$$

Then, $$I_{o,t} = A\, e^{mt} = Ae^{-t/\tau}$$

The transient response of a first-order system is same for different standard inputs. ...(2.24)

Steady state response (Particular integral) :

The steady state response is given by :

$$(1 + \tau D)\, I_{o,s} = I_i$$

(Subscript s refers to the steady state value) ...(2.25)

or, $$I_{o,s} = (1 + \tau D)^{-1}\, I_i$$

$$= (1 - \tau D + \text{terms in } D^2 \text{ and higher})\, I_i$$ ...(2.26)

1. Step input :

Since the input I_i is a step of constant magnitude; its differential equals zero, and subsequently, we get:

$$I_{o,s} = (1 - \tau D)\, I_i = I_i$$ ...(2.27)

Total response = Transient response + steady state response

or, $$I_o = Ae^{-t/\tau} + SI_i$$ ...(2.28)

The constant A is evaluated from the initial conditions as follow :

At, $$t = 0, \qquad I_o = 0$$

$\therefore$ $$0 = A + SI_i \quad \text{or,}\ A = -\,SI_i$$

$\therefore$ $$I_o = \underbrace{-I_i\, e^{-t/\tau}}_{\text{Transient}} + \underbrace{I_i}_{\text{steady state}}$$

or, $$I_o = I_i\,(1 - e^{-t/\tau})$$ ...(2.29)

or, $$\frac{I_o}{I_i} = (1 - e^{-t/\tau})$$ ...(2.30)

.... in *non-dimensional form.*

Salient features (with step input):

Following are the salient features of first-order system with step input:

(*i*) The transient response of the first-order system is *time dependent*; as the time passes, grows its value decreases (Refer to eqn. 2.30) and after a very long time the value becomes zero approximately. Thus magnitude of output (I_o) will be same as input (I_i) when the time is very large.

(*ii*) The speed of response relates to the time constant τ. A large τ indicates that response of the system is slow, whereas a small τ represents a fast system response. Thus in order to get good fidelty (*i.e.*, for accurate dynamic measurements) efforts should be made to minimise the value of τ.

(*iii*) Refer to Fig. 2.13, which shows the time response of a first-order system to a step-input when $t = \tau; \dfrac{I_o}{I_i} = (1 - e^{-1}) = 0.632$. Thus, the *time constant* (τ), *for a rising exponential function, is defined as the time to reach* 63.2% of its steady state value. The time constant, for a *decaying function* would correspond to the time taken to fall to 36.8% *of its initial value*.

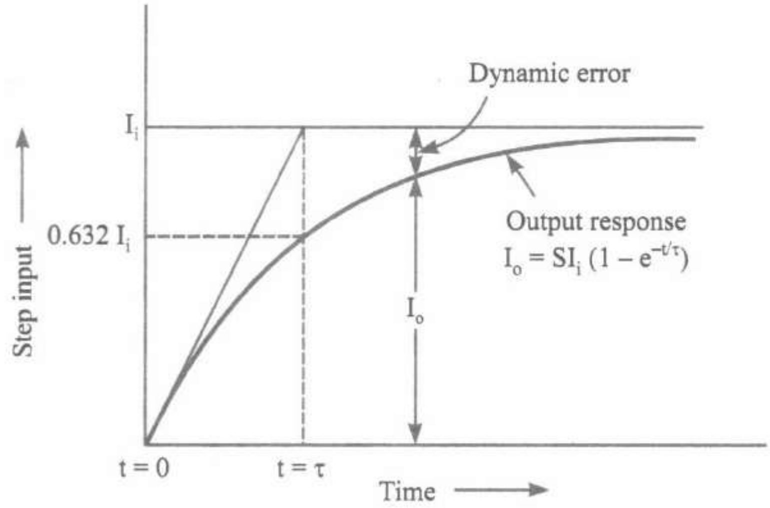


Fig. 2.13. Time response of a first-order system to step input.

(*iv*) Dynamic error (*i.e.*, vertical difference between the input and output response curve),

$$
\begin{aligned}
E_{dy.} &= I_i - I_o \\
&= I_i - I_i(1 - e^{-t/\tau}) \\
&= -I_i\, e^{-t/\tau} \qquad \text{...(2.31)}
\end{aligned}
$$

or, $$\frac{E_{dy.}}{I_i} = -e^{-t/\tau} \qquad \text{...(2.32)}$$

.... (*in dimensionless form*)

(*v*) In case the measurand has an initial value of I_{initial} at $t = 0$, then the output. I_o at any instant t is given by:

$$I_o = I_i\,[1 - e^{-t/\tau}] + I_{\text{initial}}\, e^{-t/\tau}$$

or, $$I_o = I_i + (I_{\text{initial}} - I_i)\, e^{-t/\tau} \qquad \text{...(2.33)}$$

(*vi*) The speed response of a system is defined in terms of *settling time* (it is time taken by the system to reach and remain within a certain percentage tolerance band of the final steady state value). Smaller the setling time, faster is the response. Typical value of tolerance band are 2% and 5% settling times.

For unit step input, the output reaches the values given in Table 2.1 at various intervals of time. This table also shows the per unit dynamic error at different intervals of time.

TABLE 2.1 Per unit output and dynamic error for a step input for a first-order system.

t/τ	**Per unit output (I_o / I_i)**	**Per unit dynamic error ($E_{dy.}/I_i$)**
0	0	1.000
1	0.632	0.368
2	0.865	0.135
3	0.950	0.050
4	0.982	0.018
5	0.993	0.007
∞	1.000	0

Thus, 5% settling time means that the system has reached its specified value after a time which is thrice the time constant (Table 2.1).

2. Ramp Input

Consider that the input signal varies linearly with time so that ramp input I_i is prescribed by the relation $I_i = \psi t$, where ψ is constant. Then the governing differential equation is given by:

$$(1 + \tau D)\, I_o = \psi t \qquad \text{...(2.34)}$$

Now, *Transient response* (complimentary function) is given as:

$$I_{o,t} = A\, e^{-t/\tau} \qquad \text{... as before}$$

and, *steady state response* (particular integral) will be:

$$I_{o,s} = (1 - \tau D + \text{terms in } D^2 \text{ and higher})\, \psi t = \psi t - \tau \frac{d}{dt}(\psi\tau)$$

or,
$$I_{o,s} = \psi t - \psi\tau \qquad \text{...(2.35)}$$

$\therefore$ Complete response = Transient response + steady state response

$$I_o = A\, e^{-t/\tau} + (\psi t - \psi\tau) \qquad \text{...(2.36)}$$

The value of constant A can be evaluated by applying the initial condition,

At, $t = 0 \qquad I_0 = 0$

or, $0 = A - \psi\tau \quad \therefore A = \psi\tau$

$$\therefore \quad I_o = \psi t - \psi\tau + \psi\tau \times e^{-t/\tau} = \psi(t - \tau) + \psi\tau\, e^{-t/\tau} \qquad \text{...(2.37)}$$

or,
$$I_o = \psi[t - \tau(1 - e^{-t/\tau})] \qquad \text{...(2.37a)}$$

Fig. 2.14 shows the time response of a first-order system to a ramp input.

The dynamic error,

$$E_{dy.} = I_i - I_o$$
$$= \psi t - [\psi t - \psi\tau + \psi\tau\, e^{-t/\tau}]$$
$$= \underbrace{\psi\tau}_{\text{steady}} - \underbrace{\psi\tau\, e^{-t/\tau}}_{\text{Transisent}} \qquad \text{...(2.38)}$$

or,
$$\frac{E_{dy.}}{\psi\tau} = 1 - e^{-t/\tau} \qquad \text{...(2.39)}$$

... (in dimensionless form)

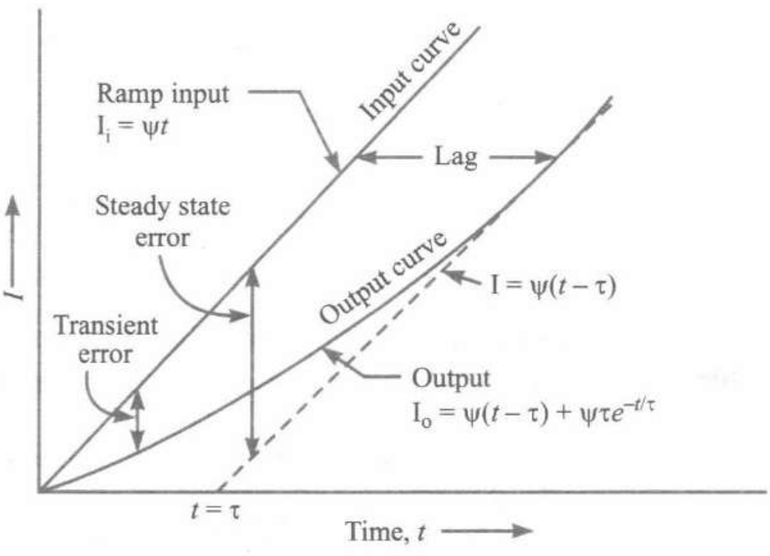


Fig. 2.14. Time response of a fist-order system to a ramp input.

Salient features (with ramp input) :

(*i*) the term $\psi\tau$ being independent of time continues to exist and so it is called the steady state error. The term $\psi\tau\ e^{-t/t}$ gradually decreases with time and hence is called the *transient error*.

- Since the steady state error is directly proportional to τ (time constant), therefore, the larger the value of t the larger will be the magnitude of the error.
- When τ is made small the transient error decreases rapidly; this implies, that the system attains the steady state at a faster pace.

(*ii*) The output response curve always lags behind the input curve by a constant amount known as **lag**.

3. Sinusoidal (Harmonic) input :

The frequency analysis of a system pertains to the steady state response of the system to a sinusoidal input. In this analysis, the system is subjected to a sinusoidal input and the system response studied with frequency as the independent variable. The sinusoid is a unique input signal, and the resulting output signal for a linear system is sinusoidal in the steady state. However, the output signal differs from the input waveform in amplitude and phase.

In order to determine the frequency response of sinusoidal input to a first-order system, let us replace the transfer operator D by a factor $j\omega$ in the input/output relationship; then we get:

$$\frac{I_o}{I_i} = \frac{1}{1+D\tau} = \frac{1}{1+j\omega\tau} \quad ...(2.40)$$

where, ω = Input frequency, rad/s, and

$j = \sqrt{(-1)}$

In a frequency response the following two quantities are of interest : Refer to Fig. 2.15.

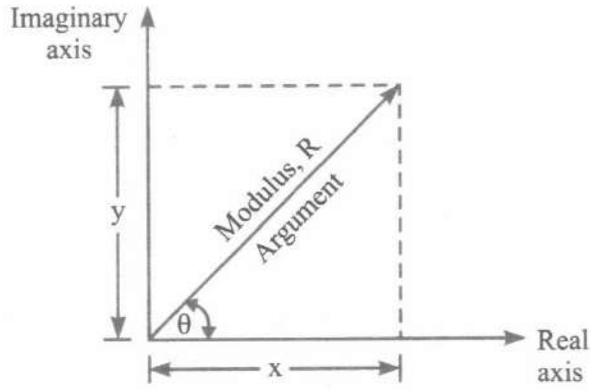


Fig. 2.15.

(*i*) *Amplitude ratio or modulus* $\left(\frac{I_o}{I_i}\right)$. It prescribes the size of the output amplitude relative to the input amplitude.

(*ii*) *Phase shift* of output relative to input.

For the first-order system represented by the equation (2.40),

$$\text{Modulus} = \sqrt{1+(\omega\tau)^2}$$

$$\text{Argument/Phase angle} = \tan^{-1}(\omega\tau) \quad ...(2.41)$$

Salient features (*with sinusoidal input*) : Refer to Fig. 2.16.

(*i*) When a system is subjected to a sinusoidal input with frequency ω, its output will also be sinusoidal, but the magnitude of the output amplitude necessarily may not be the same (as the input one). *The ratio of the amplitude* (often called *attenuation*) is given as :

$$\frac{I_o}{I_i} = \frac{1}{\sqrt{1+(\omega\tau)^2}} \quad ...(2.42)$$

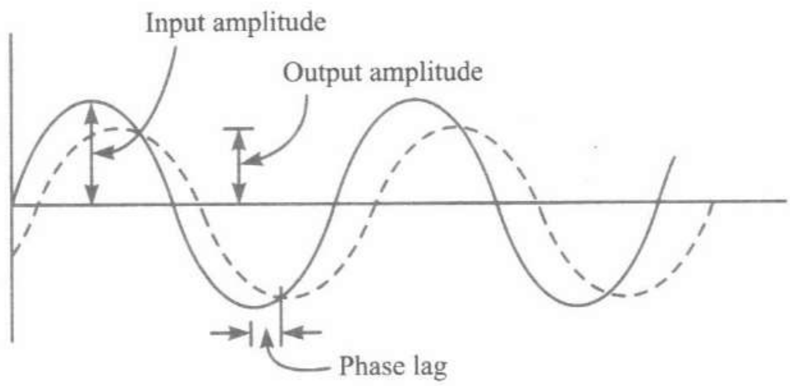


Fig. 2.16. Relationship between an input frequency and corresponding output frequency.

Thus, with the increase in input frequency, the amplitude ratio decreases.

(*ii*) The output from the system may not necessarily be in phase with the input; and the phase difference is given by:

$$\phi \text{ (phase angle) } = -\tan^{-1}(\omega\tau) \qquad ...(2.43)$$

– ve indicates that output *lags* behind the input. When $\omega = \frac{1}{\tau}$ the phase lag is $\frac{\pi}{4}$ or 45°.

As the accuracy of an instrument measuring dynamic input depends upon the time constant, therefore, smaller the time constant, greater the accuracy; for phase shift to be small, the time period τ should be small.

(*iii*) When the input and output signals are given by the relations :

$$I_i = A \sin \omega t, \text{ and } I_o = B \sin(\omega t + \phi) = zA \sin(\omega t + \phi),$$

Then the amplitude ratio may be represented as follows:

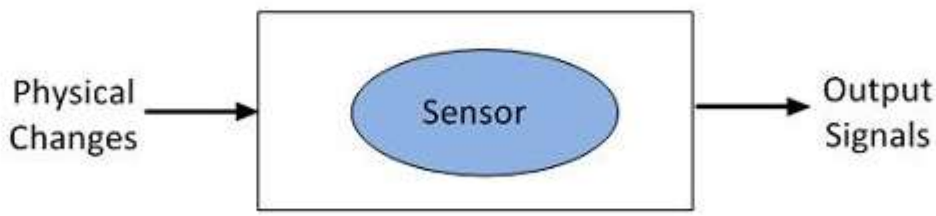
$$K = \left|\frac{I_o}{I_i}\right| = \frac{z}{\sqrt{1+(\omega\tau)^2}} \qquad ...(2.44)$$

In order to produce amplitude of sinewave without any attenuation ($K = 1$) we must use an instrument whose time constant , $\tau = \frac{\sqrt{z^2-1}}{\omega}$.

DEFINITION OF SENSOR

The sensor is a device that measures the physical quantity (i.e. Heat, light, sound, etc.) into an easily readable signal (voltage, current etc.). It gives accurate readings after calibration.

Examples – The mercury used in the thermometer converts the measurand temperature into an expansion and contraction of the liquid which is easily measured with the help of a calibrated glass tube. The thermocouple also converts the temperature to an output voltage which is measured by the thermometer.



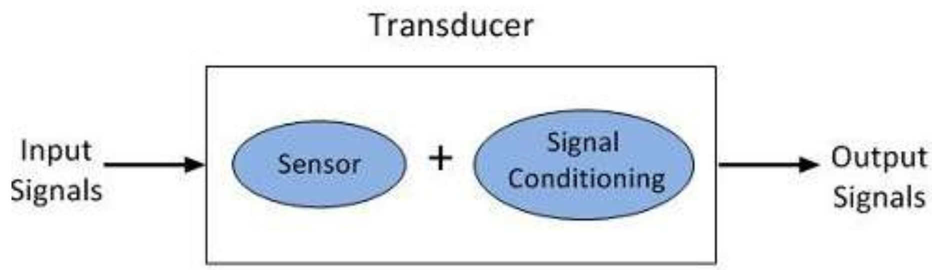
Sensor

The sensors have many applications in the electronics equipment. The few of them are explained below.

1. The motion sensors are used in the home security system and the automation door system.
2. The photo sensor senses the infrared or ultraviolet light.
3. The accelerometer sensor use in the mobile for detecting the screen rotations.

DEFINITION OF TRANSDUCER

The transducer is a device that changes the physical attributes of the non-electrical signal into an electrical signal which is easily measurable. The process of energy conversion in the transducer is known as the transduction. The transduction is completed into two steps. First by sensing the signal and then strengthening it for further processing.



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The transducer has three major components; they are the input device, signal conditioning or processing device and an output device.

The input devices receive the measurand quantity and transfer the proportional analogue signal to the conditioning device. The conditioning device modified, filtered, or attenuates the signal which is easily acceptable by the output devices.

Key Differences Between Sensor and Transducer

The following are the key differences between the sensor and transducer.

1. The sensor senses the physical change across the surrounding whereas the transducer transforms the one form of energy into another.
2. The sensor itself is the major component of the sensor, whereas the sensor and the signal conditioning are the major elements of the sensor.
3. The primary function of the sensor is to sense the physical changes, whereas the transducer converts the physical quantities into an electrical signal.
4. The accelerometer, barometer, gyroscope are the examples of the sensors whereas the thermistor, and thermocouple is the examples of the transducer

Classifications:-

1.Based upon transduction principle used

2. Primary & Secondary transducers

3.Active & Passive transducers

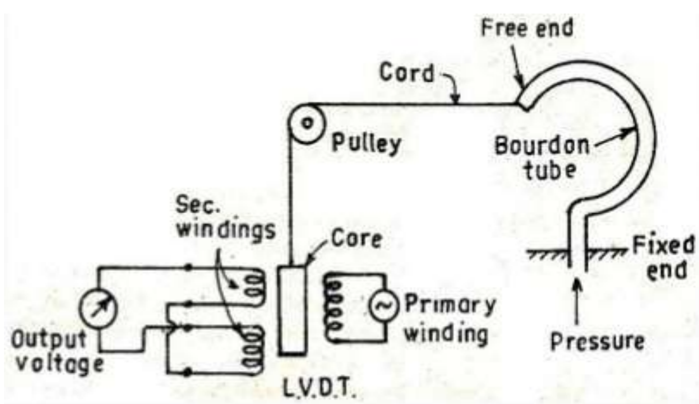
4. Analoge & Digital transducers

5.Transducers & Inverse transducers

1.Based upon the transduction principle:-

- **Resistance -** Potentimeter devices, Resistance strain gauge, Pirani guage or hot wire meter, Resistance thermometer, Thermister, Resistance hygrometer, Photoconductive cell.
- **Capacitance -** Variable capacitance pressure gauge, Capacitor microphone, Dielectric gauge.
- **Inductance -** Magnetic circuit transducer, Reluctance pick-up, Differential transformer, Eddy current guage, Magneto striction gauge.
- **Voltage & Current -** Hall effect transducer, Ionisation chamber, Photoemissive cell, Photomultiplier tube.
- **Self generating transducers -** Thermocouple, Thermopile, Moving coil generator, Piezoelectric transducer, Photovoltaic.

2. Primary & Secondary transducers:-



Look at the above picture, it is easy to understand the concept with this picture. Here the measuring variable pressure is passed through bourdon tube. Bourdon converts the pressure into displacement by the movement of free end. This displacement is transferred to iron core of LVDT. LVDT converts

displacement to voltage.
Here bourdon tube acts as primary transducer and LVDT acts as secondary transducer.

3. Active & Passive:-

Transducers those which don’t require an auxilary power source to produce their output are known as ‘Active transducers’ or self generating type.

eg: Moving coil, Piezoelectric crystal, Thermocouple, Photovoltaic cell.
On the other hand transducer that can’t work on the absence of external power supply are called ‘passive transducers’.
eg: Resistive, Capacitive, Inductive.

4. Analoge & Digital :-

Analoge transducers converts input quantity into an analog output which is continous function of time. eg: strain gauge, LVDT, Thermocouple. Thermister.

Digital transducer converts input quantity into an electrical output is in the form of pulses eg: Glass scale, Metallic scale.

5. Transducers & Inverse Transducers

We have already looked into transducers, inverse transducers are just the opposite of transducers in function. I Inverse transducers converts electrical quantity in non electrical quantity. Just think about an current coil moving in magnetic field which is an inverse transducer.
eg: Peizoelectrical crystal.

SELECTION OF A TRANSDUCER

Transducers are often employed at the boundaries of automation, measurement, and control systems. Before choosing a transducer we have to ensure that the transducer is suitable for our need. These are factors which may influence the selection.

Operating principle:

There are operating principles such as resistive, inductive, capacitive, piezoelectric, photo-voltaic, ionization etc.,

.

Sensitivity:

Transducer must be sensitive enough not to give an output, but to give an detectable output.

Operating range:

A transducer should have good resolution over it's entire range of operation. We cannot choose a temperature sensing transducer that work in range 0-100 degree Celsius for a use in boiler where temperature is up to 1000 degree Celsius, that's what it means.

Accuracy:

High degree of accuracy is needed and small value of repeatablity.

Cross-sensitivity:

Cross-sensitivity must taken into account while measuring mechanical quantity. Cross-sensitivity must reduced. Because when transducer subjected to measure variations in one plain, while actual quantity to be measured is in another plain.

Error:

A transducer must avoid error. For which a transducer should maintain expected input output relationship by it's transfer function.

Transient & Frequency response:

Transducer should meet desired time domain specifications such as peak over shoot, raise time, setting time & small dynamic error, it should flat frequency response curve with higher cut-off frequency at high limit in-order to have a high value.

Loading effect:

Transducer should have high input impedance and low output impedance to avoid loading effect.

Environment compatibility:

Under specified environment conditions the transducer maintain input-output relationship and doesn't breakdown. Transducer should be able to withstand temperature, pressure shock etc., when subjected to it, if it is application require so.

Insensitivity to unwanted signals:

Noise should be avoided in measuring, so that a good transducer should insensitive to unwanted signals. But high sensitive to desired signals.

Usage and ruggedness:

Ruggedness both of mechanical and electrical intensities of transducer VS it's size and weight must be considered while selection.

STRAIN GAUGE MEASURMENTS

Introduction:

- When a metal conductor is stretched or compressed, its resistance changes an account of the fact that both length and diameter of conductor change.
- The value of resistivity of the conductor also changes. When it is strained its property is called piezo- resistance.
- Therefore, resistance strain gauges are also known as piezo-resistive gauges.

Strain gauge:

- A strain gauge is a device which is used to measure dimensional change on the surface of a structural member under test.
- Strain gauges give indication of strain at only one point.

Types of strain gauge:

Four types of strain gauges are:

- Wire-wound strain gauge
- Foil-type strain gauges.
- Semiconductor strain gauges

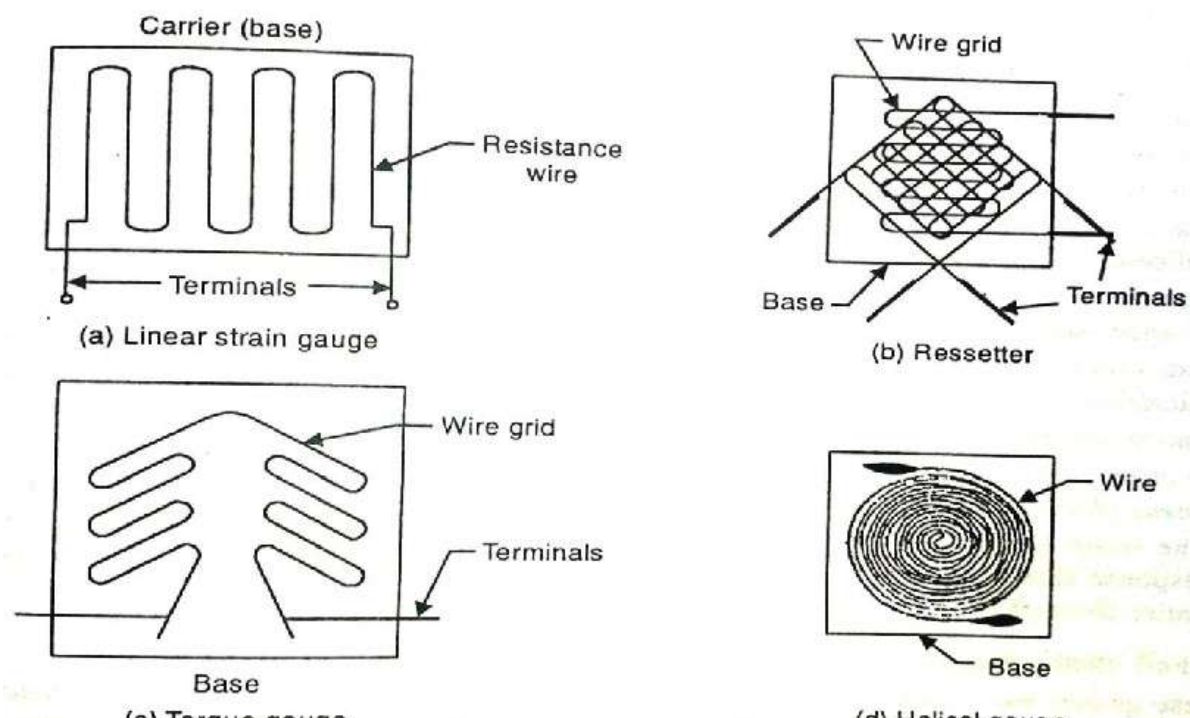
1. Wire wound strain gauges:

These are two main classes of wire-wounded strain gauges:

- Bonded strain gauge.
- Unbounded strain gauge.

Bounded strain gauge:

- It is composed of fine wire, wound and cemented on a resilient insulating support, usually a wafer unit.



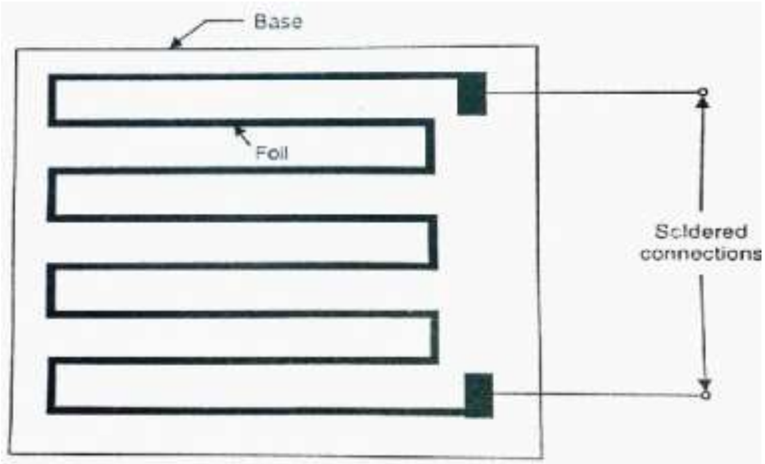
Resistance wire strain gauges.

- Units may be mounted upon or incorporated In mechanical elements whose deformations under stress are to be determined.

While there are no limits to the basic values which may be selected for strain-gauge resistance

2. Foil strain gauges:

In these gauges the strain is sensed with help of metal foil. Foil gauges have a much greater dissipation capacity as compared with wire wounded gauges on account of their greater surface area for the same volume. Due to this reason they can be employed for high operating temperature range.



Foil gauge

In these gauges, the bonding is better due to large surface area of the foil. The bonded foil gauges find a wider field of action.

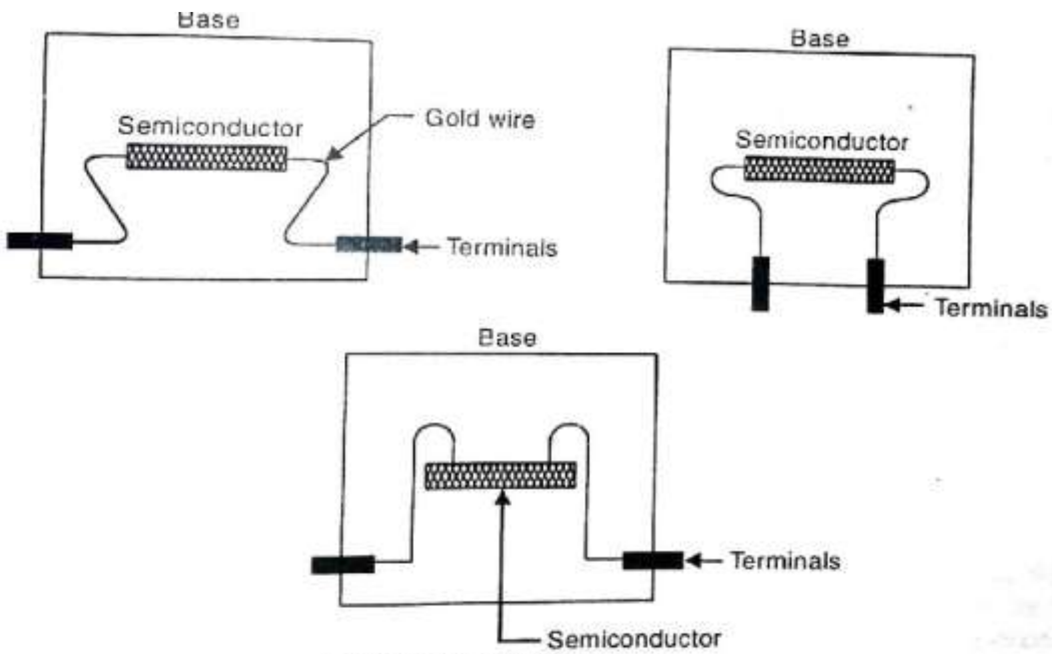
The characteristics of foil type's strain gauges are similar to those of wire wounded strain gauges and their gauge factors are typically the same as that of wire wounded strain gauges.

3. Semiconductor strain gauges:

- Semiconductor strain gauges depend for their action upon piezo-resistive effect i.e., the change in value of the resistance due to change in resistivity.
- These gauges are used where a very high gauge factor and small envelope are required.
- For semiconductor strain gauges semiconducting materials such as silicon and germanuium are used.
- A typical strain gauge consists of a strain sensitive crystal material and leads that are sandwiched in a protective matrix.

The production of these gauges employs conventional semiconductor technology using semiconducting filaments which have a thickness of 0.05 mm and bonding them on suitable insulating substances, such as Teflon

The resistance value of foil gauges which are commercially available is between 50 and 1000 Ω.



Semiconductor strain gauges.

- Gold, leads are generally applied for making the contacts.

Advantages:

- These gauges have high gauge factor.
- Excellent hysteresis characteristics.

Disadvantages:

- Linearity of these gauges is poor.
- Manometers are used up to 200 kN/m^2; above this pressure dead weight testers are used.

CHARACTERISTICS OF STRAIN GAUGES

The characteristics of strain gauges are as follows:

1. They are highly precise and don't get influenced due to temperature changes. However, if they do get affected by temperature changes, a thermistor is available for temperature corrections.
2. They are ideal for long distance communication as the output is an electrical signal.
3. Strain Gauges require easy maintenance and have a long operating life.
4. The production of strain gauges is easy because of the simple operating principle and a small number of components.
5. The strain gauges are suitable for long-term installation. However, they require certain precautions while installing.
6. All the strain gauges produced by Encardio-Rite are hermetically sealed and made up of stainless steel thus, waterproof.
7. They are fully encapsulated for protection against handling and installation damage.
8. The remote digital readout for strain gauges is also possible.

GAUGE FACTOR:

The gauge factor is defined as the unit change in resistance per unit change in length.

It is denoted as K or S. It is also called sensitivity of the strain gauge.

$$S = \frac{\Delta R/R}{\Delta l/l}$$

S = Gauge factor or sensitivity

R = Gauge wire resistance

ΔR = Change in wire resistance

l = Length of the gauge wire in unstressed condition

Δl = Change in length in stressed condition.

Derivation: Consider that the resistance wire is under tensile stress and it is deformed by ~l as shown in the Fig.

When uniform stress (J is applied to this wire along the length, the resistance R

Let ρ = Specific resistance of wire material in Ω-m

l = Length of the wire in m

A = Cross-section of the wire in m^2

changes to R + ~R because of change in length and cross-sectional area.

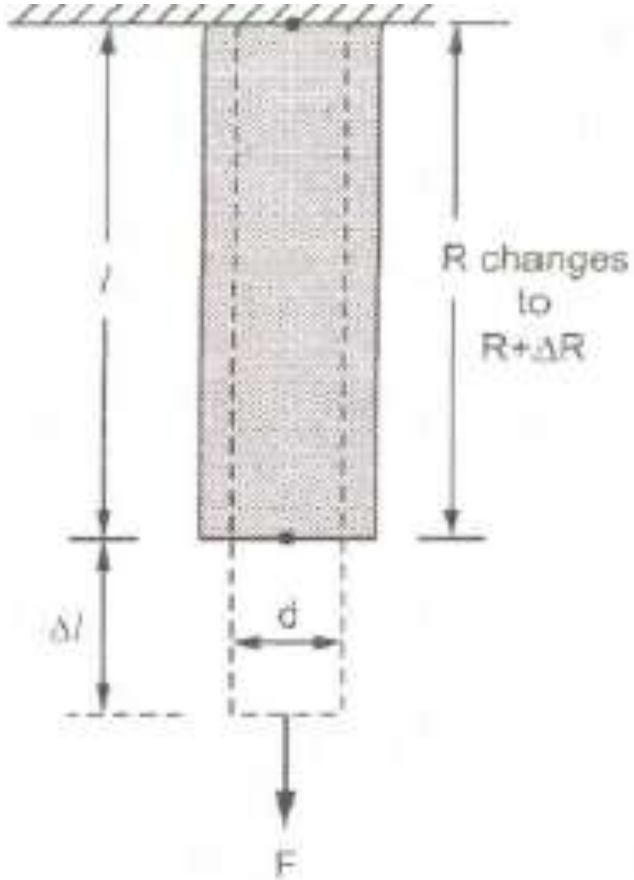
$$\sigma = \text{Stress} = \frac{\Delta l}{l}$$

$\Delta l/l$ = Per unit change in length

$\Delta A/A$ = Per unit change in area

$\Delta \rho/\rho$ = Per unit change in resistivity (specific resistance)

$$R = \frac{\rho l}{A}$$



$$\therefore \quad \frac{dR}{d\sigma} = \frac{d\left(\frac{\rho l}{A}\right)}{d\sigma} = \frac{\rho}{A}\frac{\partial l}{\partial \sigma} - \frac{\rho l}{A^2}\frac{\partial A}{\partial \sigma} + \frac{l}{A}\frac{\partial \rho}{\partial \sigma}$$

Note that $\quad \frac{\partial}{\partial \sigma}\left(\frac{1}{A}\right) = -\frac{1}{A^2}\frac{\partial A}{\partial \sigma}$

Multiply both sides by $\frac{1}{R}$,

$$\frac{1}{R}\frac{dR}{d\sigma} = \frac{\rho}{RA}\frac{\partial l}{\partial \sigma} - \frac{1}{R}\frac{\rho l}{A^2}\frac{\partial A}{\partial \sigma} + \frac{l}{RA}\frac{\partial \rho}{\partial \sigma}$$

Using $\quad R = \frac{\rho l}{A}$ on right hand side,

$$\frac{1}{R}\frac{dR}{d\sigma} = \frac{1}{l}\frac{\partial l}{\partial \sigma} - \frac{1}{A}\frac{\partial A}{\partial \sigma} + \frac{1}{\rho}\frac{\partial \rho}{\partial \sigma}$$

Canceling $\partial\sigma$ from both sides,

$$\frac{dR}{R} = \frac{dl}{l} - \frac{dA}{A} + \frac{\partial\rho}{\rho}$$

i.e.

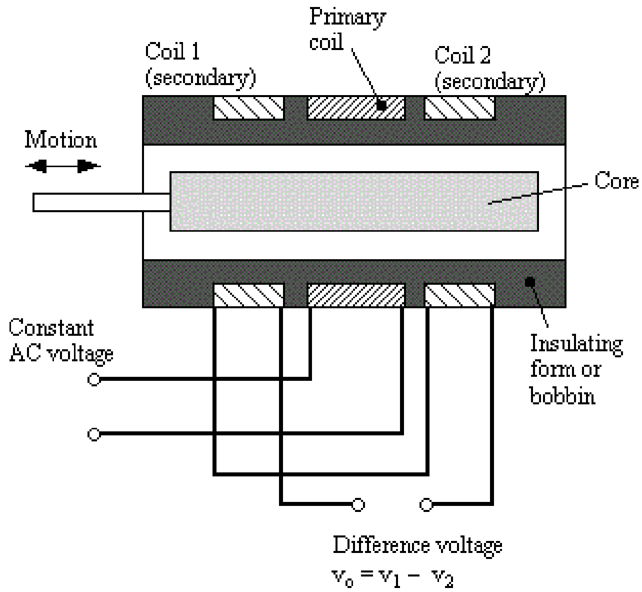
$$\frac{\Delta R}{R} = \frac{\Delta l}{l} - \frac{\Delta A}{A} + \frac{\Delta\rho}{\rho}$$

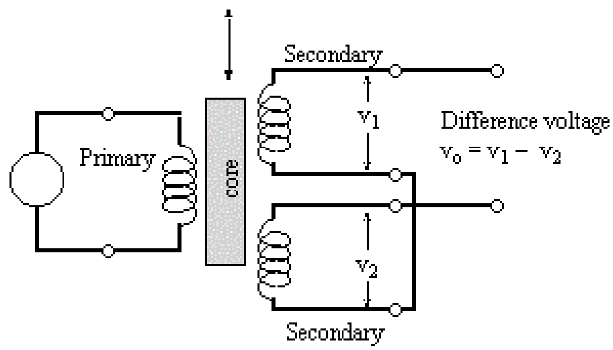
LINEAR VARIABLE DIFFERENTIAL TRANSFORMER (LVDT)

Linear Variable Differential Transformer, LVDT is the most used inductive transducer for translating linear motion into electrical signal. This transducer converts a mechanical displacement proportionally into electrical signal.

Main Features of Construction

- The transformer consists of a primary winding P and two secondary windings S_1 and S_2 wound on a cylindrical former (which is hollow in nature and contains the core).
- Both the secondary windings have an equal number of turns, and we place them on either side of primary winding
- The primary winding is connected to an AC source which produces a flux in the air gap and voltages are induced in secondary windings.
- A movable soft iron core is placed inside the former and displacement to be measured is connected to the iron core.
- The iron core is generally of high permeability which helps in reducing harmonics and high sensitivity of LVDT.
- The LVDT is placed inside a stainless steel housing because it will provide electrostatic and electromagnetic shielding.
- The both the secondary windings are connected in such a way that resulted output is the difference between the voltages of two windings.



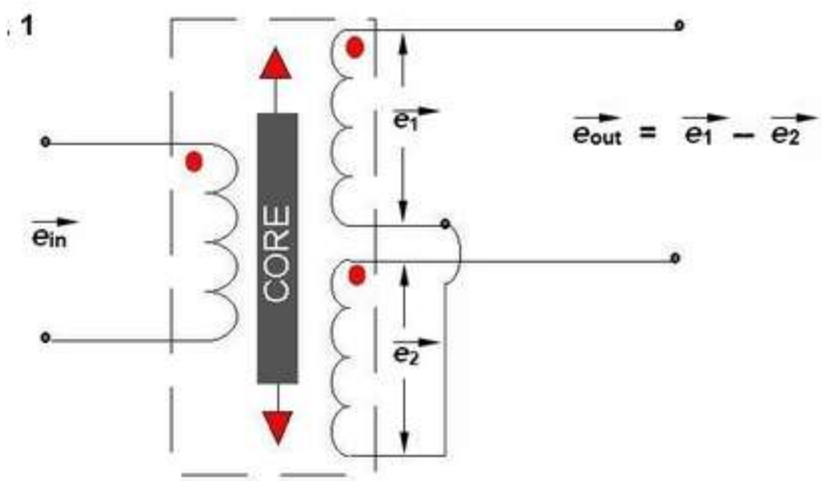


Principle of Operation and Working

As the primary is connected to an AC source so alternating current and voltages are produced in the secondary of the LVDT. The output in secondary S_1 is e_1 and in the secondary S_2 is e_2. So the differential output is,

$$e_{out} = e_1 - e_2$$

This equation explains the **principle of Operation of LVDT**.



Now three cases arise according to the locations of core which explains the working of LVDT are discussed

- CASE I When the core is at null position (for no displacement)

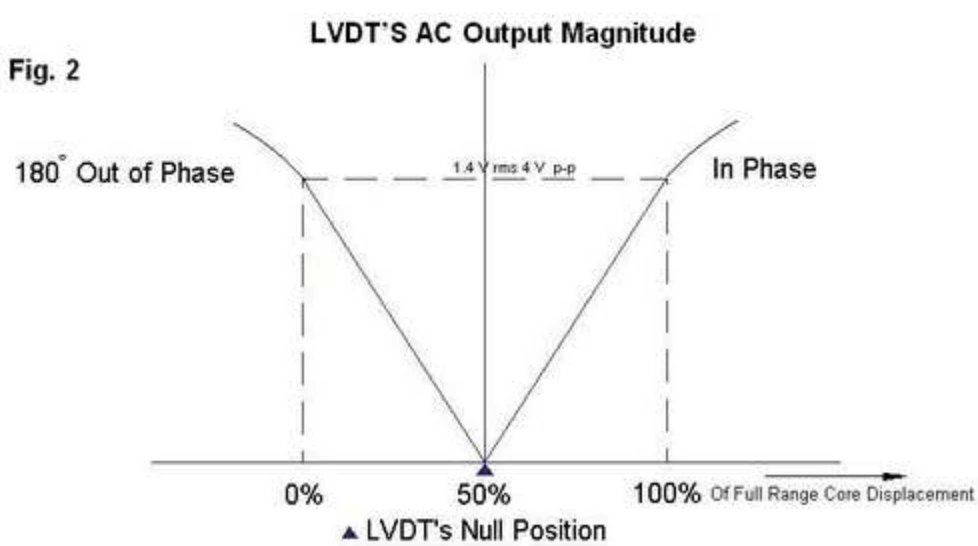
  When the core is at null position then the flux linking with both the secondary windings is equal so the induced emf is equal in both the windings. So for no displacement the value of output e_{out} is zero as e_1 and e_2 both are equal. So it shows that no displacement took place.

- CASE II When the core is moved to upward of null position (For displacement to the upward of reference point)

  In the this case the flux linking with secondary winding S_1 is more as compared to flux linking with S_2. Due to this e_1 will be more as that of e_2. Due to this output voltage e_{out} is positive.

- CASE III When the core is moved to downward of Null position (for displacement to the downward of the reference point). In this case magnitude of e_2 will be more as that of e_1. Due to this output e_{out} will be negative and shows the output to downward of the reference point.

Output V_S Core Displacement A linear curve shows that output voltage varies linearly with displacement of core.



AC Output of Conventional LVDT Versus Core Displacement

Some important points about magnitude and sign of voltage induced in LVDT

- The amount of change in voltage either negative or positive is proportional to the amount of movement of core and indicates amount of linear motion.
- By noting the output voltage increasing or decreasing the direction of motion can be determined
- The output voltage of an LVDT is linear function of core displacement .

Advantages of LVDT

- High Range – The LVDTs have a very high range for measurement of displacement.they can used for measurement of displacements ranging from 1.25 mm to 250 mm
- No Frictional Losses – As the core moves inside a hollow former so there is no loss of displacement input as frictional loss so it makes LVDT as very accurate device.
- High Input and High Sensitivity – The output of LVDT is so high that it doesn't need any amplification. The transducer posseses a high sensitivity which is typically about 40V/mm.
- Low Hysteresis – LVDTs show a low hysteresis and hence repeatability is excellent under all conditions
- Low Power Consumption – The power is about 1W which is very as compared to other transducers.
- Direct Conversion to Electrical Signals – They convert the linear displacement to electrical voltage which are easy to process

Disadvantages of LVDT

- LVDT is sensitive to stray magnetic fields so it always requires a setup to protect them from stray magnetic fields.
- LVDT gets affected by vibrations and temperature.

It is concluded that they are advantageous as compared than any other inductive transducer.

Applications of LVDT

1. We use LVDT in the applications where displacements to be measured are ranging from a fraction of mm to few cms. The **LVDT** acting as a primary transducer converts the displacement to electrical signal directly.
2. The LVDT can also act as a secondary transducer. E.g. the Bourbon tube which acts as a primary transducer and it converts pressure into linear displacement and then LVDT coverts this displacement into an electrical signal which after calibration gives the readings of the pressure of fluid.

ROTARY VARIABLE DIFFERENTIAL TRANSFORMER (RVDT)

Definition: The transformer which **senses** the **angular displacement** of the conductor is **known** as the **Rotary Variable Differential Transformer** or RVDT. It is the type of electromechanical **transducer which** gives **the** linear output proportional **to the** input angular displacement**.**

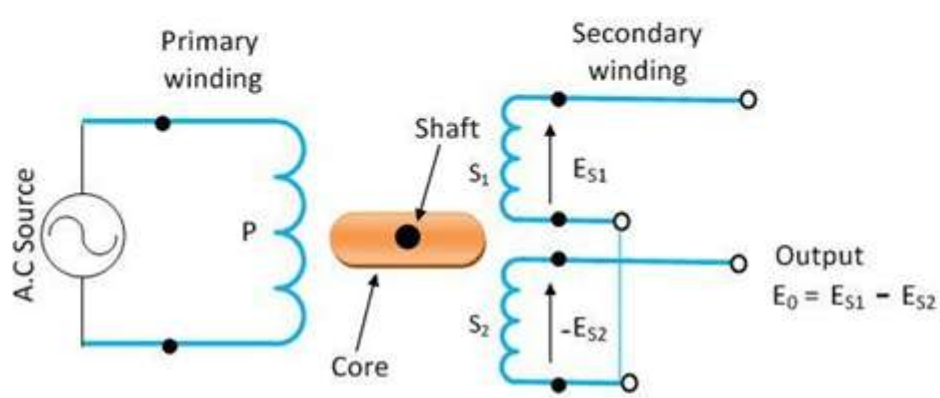
RVDT CONSTRUCTION AND ITS WORKING

RVDT transducer has two windings similar to a normal transformer such as primary winding and two secondary windings shown in the following **RVDT diagram**. The two windings of the transformer wounded, where the two secondary windings have an equivalent number of windings. These are located on both sides of the primary winding of the transformer. A cam formed a magnetic core which is made with a soft iron is coupled to a shaft. Thus, this core can be twisted among the windings. The construction of both the RVDT and LVDT are similar but the main difference is the shape of the core in transformer windings. This core will turn between the two windings of the transformer due to the shaft.

RVDT Construction

The typical RVDTs are linear over a +40 or -40 degrees, Sensitivity is about 2mV to 3mV per degree of rotation and the input voltage range is 3V RMS at frequency ranges from 400Hz to 20kHz. Based on the movement of the shaft in the transformer, the three conditions will be produced such as

- When the Core is at Null Position
- When the Core Rotates in Clockwise Direction
- When the Core Rotates in Anticlockwise Direction



When the Core is at Null Position

In the first condition, when the shaft is placed at the null position then the induced e.m.f in the secondary windings are similar although reverse in phase. Thus, the differential o/p potential will be zero, and the condition will be E1 = E2, where E0 = E1-E2 =0

When the Core Rotates in Clockwise Direction

In the second condition, when the shaft rotates in the direction of clockwise; more section of the core will enter across the primary winding. Therefore, the induced e.m.f across the primary winding is higher than secondary winding. Hence, the differential o/p potential is positive, and the condition will be E1 > E2, where E0 = E1-E2 = positive.

When the Core Rotates in Anticlockwise Direction

In the third condition, when the shaft rotates in the direction of anticlockwise, more section of the core will be entered across the secondary winding. Thus, the induced e.m.f across the secondary coil is higher than the primary coil. Hence, the differential o/p potential is negative that means 1800 phase shift, and the condition will be E1 < E2, where E0 = E1-E2 = negative.

RVDT Advantages and Disadvantages

The advantages of RVDT include the following.

- The consistency of RVDT is high
- The exactness of RVDT is high
- The lifespan is long
- The performance is repeatable
- The construction is compact and strong
- Durability
- Low cost
- Easy to handle electronic components

The disadvantages of RVDT mainly include the following

- The contact among the measuring exterior as well as the nozzle is not possible for all time.
- The output of the RVDT is linear (about +40 or -40 degrees), so it restricts the usability.

RVDT Applications

The applications of RVDT include the following.

- Fuel Valves as well as Hydraulic
- Modern machine tools
- Controls Cockpit
- Controls Fuel
- Brake with cable systems
- Engines bleed air-systems
- Robotics

PIEZO-ELECTRIC TRANSDUCER

Definition: The Piezoelectric transducer is an **electroacoustic transducer** use for **conversion** of **pressure** or mechanical stress into an alternating **electrical force.** It is used for measuring the physical quantity like force, pressure, stress, etc., which is directly not possible to measure.

The word piezoelectric means the electricity produces by the pressure. The Quartz is the examples of the natural piezoelectric crystals, whereas the Rochelle salts, ammonium dehydration, phosphate, lithium sulphate, dipotassium tartrate are the examples of the man made crystals. The ceramic material is also used for piezoelectric transducer.

The ceramic material does not have the piezoelectric property. The property is developed on it by special polarizing treatment. The ceramic material has several advantages. It is available in different shapes and sizes. The material has the capability of working at low voltages, and also it can operate at the temperature more than 3000ºC

Piezoelectric Effect

The EMF develops because of the displacement of the charges. The effect is changeable, i.e. if the varying potential applies to a piezoelectric transducer, it will change the dimension of the material or deform it. This effect is known as the piezoelectric effect.

The pressure is applied to the crystals with the help of the force summing devices for examples the stress is applied through mechanical pressure gauges and pressure sensors, etc. The deformation induces the EMF which determines the value of applied pressure.

Theory of Piezo-Electric Transducer

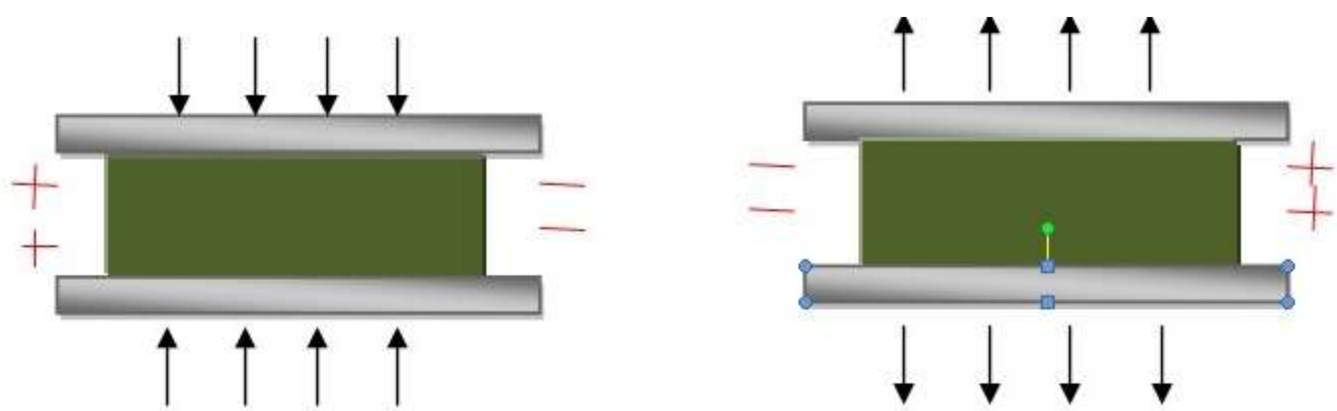
Piezoelectric Transducer Working

Piezoelectric Transducer works with the principle of piezoelectricity. The faces of piezoelectric material, usual quartz, is coated with a thin layer of conducting material such as silver. When stress has applied the ions in the material move towards one of the conducting surface while moving away from the other. This results in the generation of charge. This charge is used for calibration of stress. The polarity of the produced charge depends upon the direction of the applied stress. Stress can be applied in two forms as C**ompressive stress** and **Tensile stress** as shown below.

Working of a Piezoelectric Transducer

Piezoelectric Transducer Formula

The orientation of the crystal also effects the amount of voltage generated. Crystal in a transducer can be arranged in **longitudinal position** or **transverse position**.



Piezoelectric Transducer Formula

Longitudinal and Transverse Effect

In the longitudinal effect, the charge generated is given by

$$Q = F * d$$

Where F is the applied force,d is the piezoelectric coefficient of the crystal.

Piezoelectric coefficient d of quartz crystal is around $2.3 * 10^{-12}$ C/N.
In the transverse effect, the charge generated is given by

$$Q = F * d * (b/a)$$

When the ratio b/a is greater than 1 the charge produced by transverse arrangement will be greater than the amount generated by longitudinal arrangement.

Properties of Piezo Electric-Crystal

The following are the properties of the Piezoelectric Crystals.

1. The piezoelectric material has high stability.
2. It is available in various shapes and sizes.
3. The piezoelectric material has output insensitive to temperature and humidity.

Uses of Piezoelectric Crystal

The following are the uses of the Piezoelectric transducers.

1. The piezoelectric material has high stability and hence it is used for stabilizing the electronic oscillator.
2. The ultrasonic generators use the piezoelectric material. This generator is used in SONAR for underwater detection and in industrials apparatus for cleaning.
3. It is used in microphones and speakers for converting the electric signal into sound.
4. The piezoelectric material is used in electric lighter.

The transducer has low output, and hence external circuit is associated with it.

WHAT IS PHOTOELECTRIC TRANSDUCER?

The photoelectric transducer can be defined as, a transducer which changes the energy from the light to electrical. It can be designed with the semiconductor material.

The photoelectric transducer converts the light energy into electrical energy. It is made of semiconductor material. The photoelectric transducer uses a photosensitive element, which ejects the electrons when the beam of light absorbs through it.

This transducer utilizes an element like photosensitive which can be used for ejecting the electrons as the light beam soak ups through it. The electron discharges can change the photosensitive element's property. Therefore the flowing current stimulates within the devices. The flow of the current's magnitude can be equivalent to the whole light absorbed with the photosensitive element.

The diagram of the photoelectric transducer is shown below. This transducer soak ups the light radiation which drops over the semiconductor material. The light absorption can boost the electrons in the material, & therefore the electrons begin to move. The electron mobility can generate three effects like

- The material resistance will be changed.
- The semiconductor's o/p current will be changed.
- The semiconductor's o/p voltage will be changed.

Photoelectric Transducer Classification

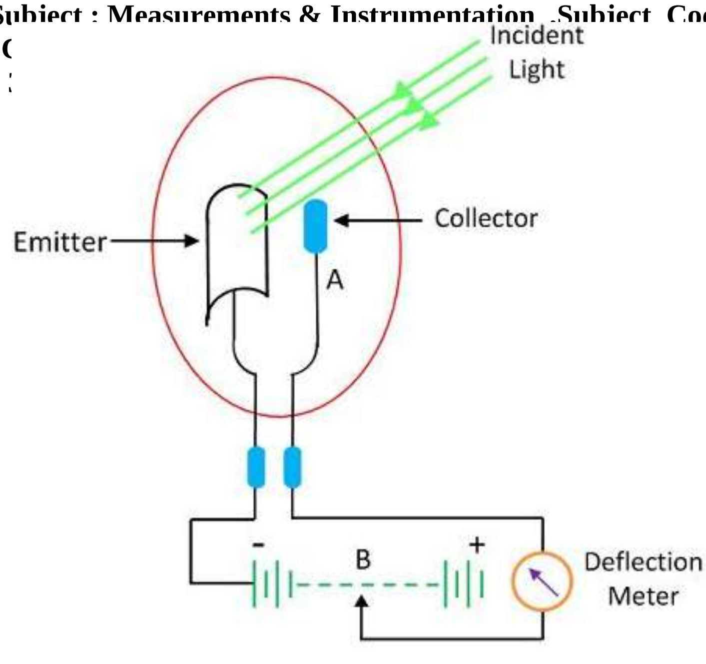
- Photo emissive Cell
- Photoconductive Cell
- Photo-voltaic cell

Classification of Photoelectric Transducers

The photoelectric transducers are classified into following ways.

1. PHOTOEMISSIVE CELL

The Photoemissive cell converts the photons into electric energy. It consists the anode rode and the cathode plate. The anode and cathode are coated with a Photoemissive material called caesium antimony.



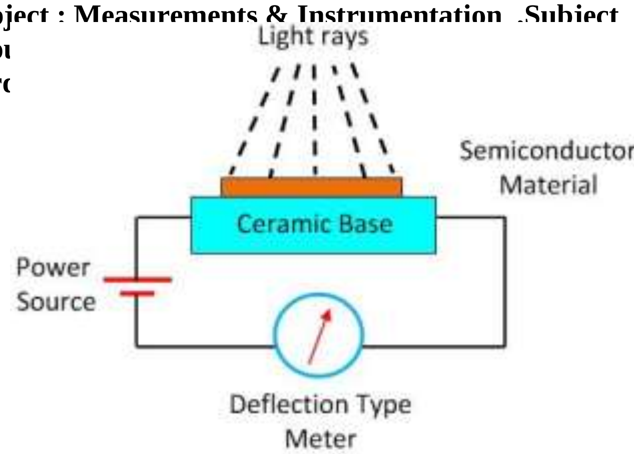
Photoemissive Cell

When the radiation of light fall on cathode plates the electrons starts flowing from anode to cathode. Both the anode and the cathode are sealed in a closed, opaque evacuated tube. When the radiation of light fall on the sealed tube, the electrons starts emitting from the cathode and moves towards the anode.

The anode is kept to the positive potential. Thus, the photoelectric current starts flowing through the anode. **The magnitude of the current is directly proportional to the intensity of light passes through it.**

2. PHOTOCONDUCTIVE CELL

The photoconductive cell converts the light energy into an electric current. It uses the semiconductor material like cadmium selenide, Ge, Se, as a photo sensing element.

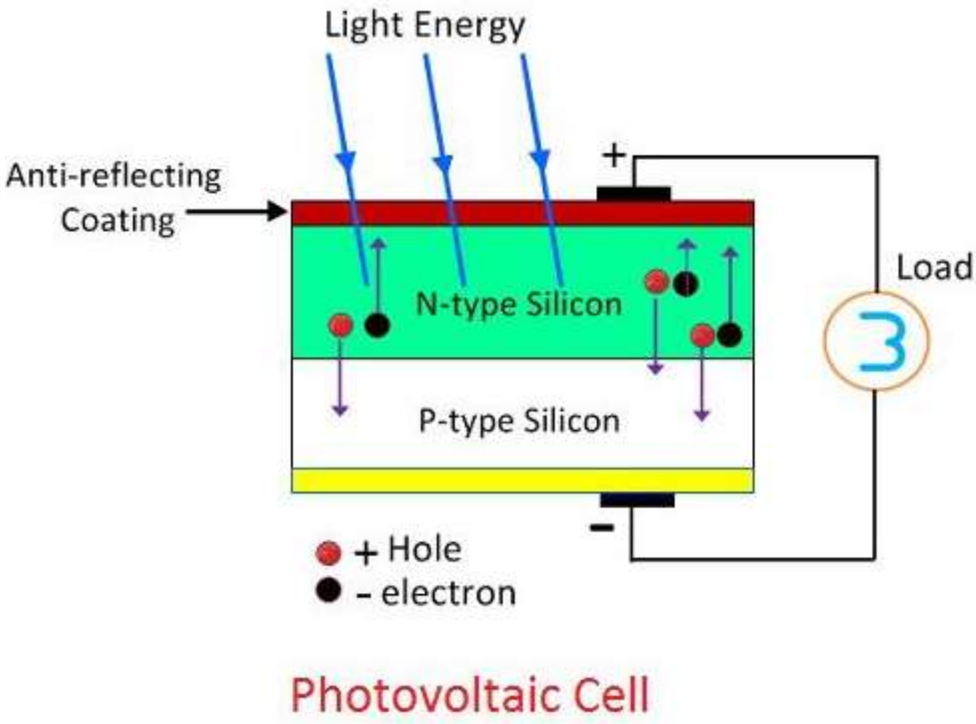


Photoconductive Cell

When the beam of light falls on the semiconductor material, their conductivity increases and the material works like a closed switch. The current starts flowing into the material and deflects the pointer of the meter.

3. PHOTO-VOLTAIC CELL

The photovoltaic cell is the type of active transducer. The current starts flowing into the photovoltaic cell when the load is connected to it. The silicon and selenium are used as a semiconductor material. When the semiconductor material absorbs heat, the free electrons of the material starts moving. This phenomenon is known as the photovoltaic effect.

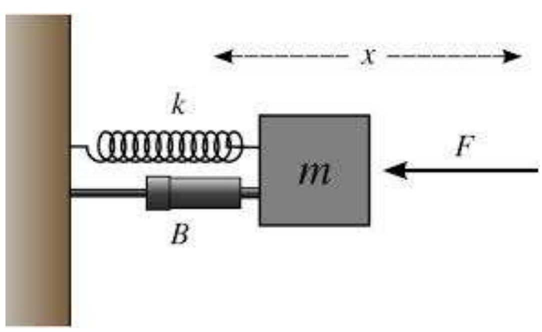


The movements of electrons develop the current in the cell, and the current is known as the photoelectric current.

MECHANICAL, HYDRAULIC & PNUMATICS AMLIFYING ELEMENTS

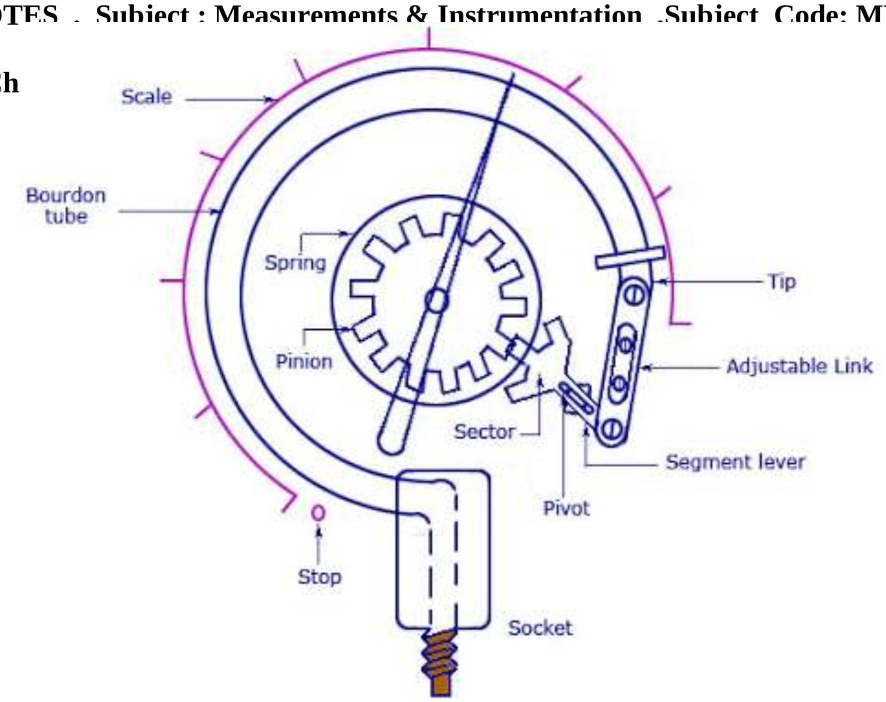
1.MECHANICAL AMPLIFYING ELEMENTS

A **mechanical** amplifier, or a **mechanical amplifying element**, is a linkage mechanism that **amplifies** the magnitude of **mechanical** quantities such as force, displacement, velocity, acceleration and torque in linear and rotational systems.



Spring Mass System

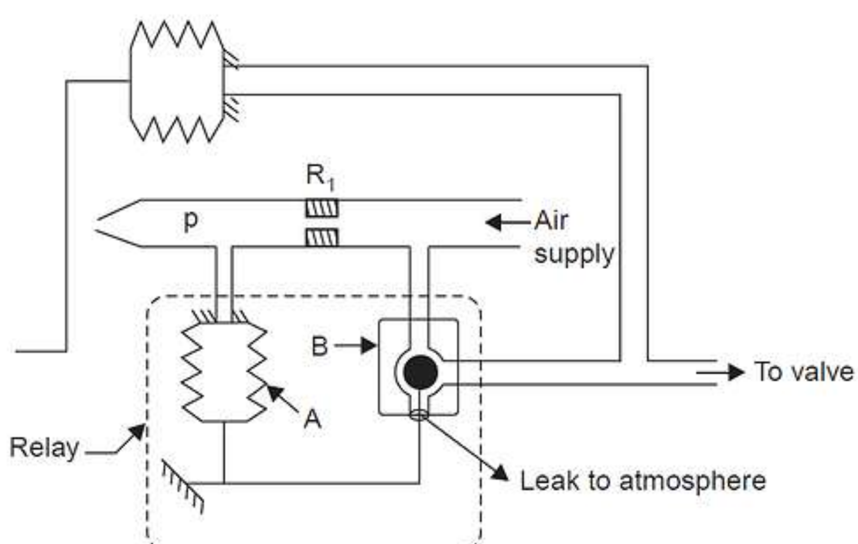
Ch



Bourdon Tube Pressure Gauge

In Bourdon tube pressure gauge Pinion , sector , C-clamp, Pointer, Link act as Mechanical amplifying Elements.

(2) PNEUMATIC AMPLIFYING ELEMENTS: A simple pneumatic relay being used as an amplifying element is shown below. These elements are used where abundant supply of compressed air is available, e.g. in factories and workshop



Pneumatic Relay working:

A power amplifier is needed to act on a large control valve. A pneumatic relay is a power amplifier. The relay is composed of small volume A bellows and a specially designed valve. In the nozzle pressure is transmitted to bellows A and, as the pressure increases, the bellows expand the ball down the valve B cavity, decreasing the leakage to the atmosphere and increasing the pressure Pv.

The relay is made in such a way that the output pressure Pv is proportional to the signal pressure p. The airflow of the relay will be much greater than the flow rate through the nozzle, since the resistance to the air supply may be too small compared to the resistance Rr in the nozzle. A relay valve can be designed as direct acting, in which case the outlet pressure is directly proportional to the inlet pressure, or it can be reverse acting, in which case the outlet pressure is inversely proportional to the inlet pressure.

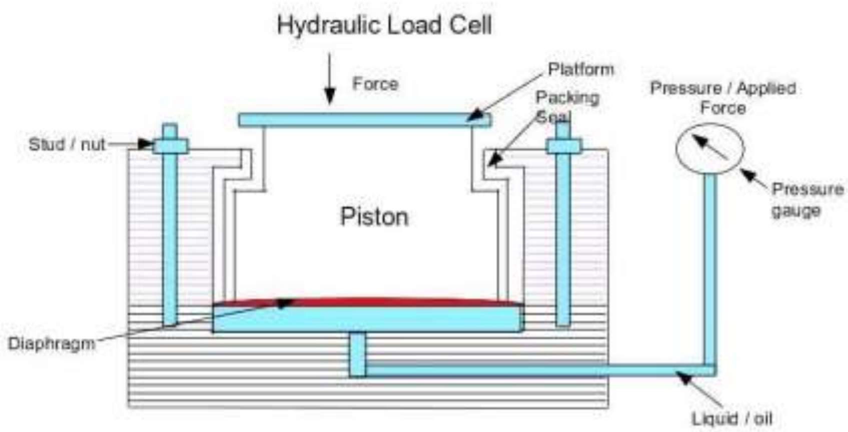
3. HYDRAULIC AMPLIFYING ELEMENTS

Hydraulic load cells are force-balance devices, measuring weight as a change in pressure of the internal filling fluid. In **hydraulic load cell**, a **load** or force acting on a loading head is transferred to a piston that in turn compresses a filling fluid confined within an elastomeric diaphragm chamber.

Principle of Hydraulic Load cell

When a force is applied on a liquid medium contained in a confined space, the pressure of the liquid increases. This increase in pressure of the liquid is proportional to the applied force. Hence a measure of the increase in pressure of the liquid becomes a measure of the applied force when calibrated.

Hydraulic Load Cell



The main parts of a hydraulic load cell are as follows

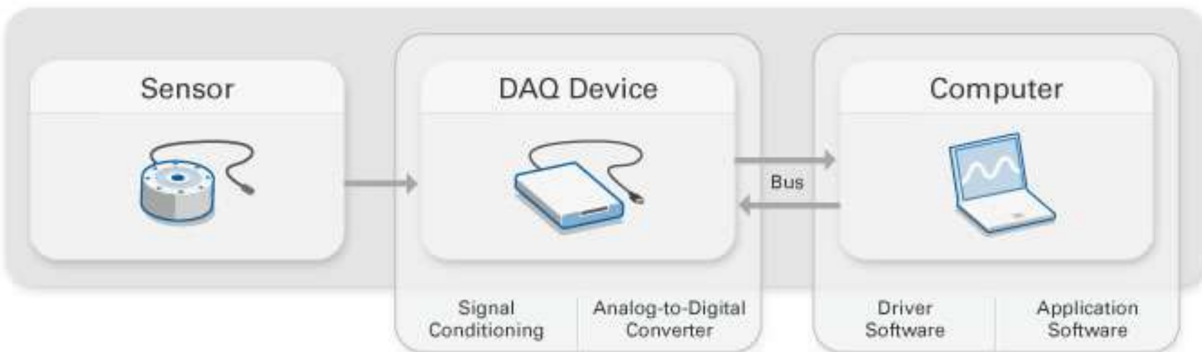
- A dirphragm
- A piston with a loading platform (as shown in figure) placed on top of the diaphragm.
- A liquid medium which is under a pre-loaded pressure is on the other side of the diaphragm.
- A pressure gauge (bourdon tube type) connected to the liquid medium.

Operation of Hydraulic Load Cell

- The force to be measured is applied to the piston.
- The appilied force moves the piston downwards and deflects the diaphragm and this deflection of the diaphragm increases the pressure in the liquid medium (oil).
- This increase in pressure of the liquid medium is proportional to the applied force. The increase in pressure is measured by the pressure gauge which is connected to the liquid medium.
- The pressure is calibrated in force units and hence the indication in the pressure gauge becomes a measure of the force applied on the piston.

DATA ACQUISITION SYSTEM

Data acquisition (DAQ) is the process of measuring an electrical or physical phenomenon such as voltage, current, temperature, pressure, or sound with a computer. A DAQ system consists of sensors, DAQ measurement hardware, and a computer with programmable software. Compared to traditional measurement systems, PC-based DAQ systems exploit the processing power, productivity, display, and connectivity capabilities of industry-standard computers providing a more powerful, flexible, and cost-effective measurement solution.



Data acquisition system (DAS)

Data acquisition system (DAS) is a computerized system that collects data from the real world, converts it into the form of electrical signals and do required processing on it for storage, and presentation on computers.

The complete system is controlled and operated by a software application. This software application is developed by using general-purpose high-level programming languages like C, C^{++}, java, etc.

These systems are used in industrial and commercial fields. They are used for collecting, storing and processing of data.

The data acquisition system can be divided into two types:

- Analog data acquisition system
- Digital data acquisition system

The analog data acquisition system gives an analog output whereas the digital data acquisition system gives a digital output. Analog DAS is used when wide frequency width is required or when lower accuracies can be tolerated.

Digital DAS is used when physical quantity being monitored has a narrow bandwidth (i.e. when the quantity varies slowly). Also, high accuracy and low per channel cost are required. These are more complex than analog DAS.

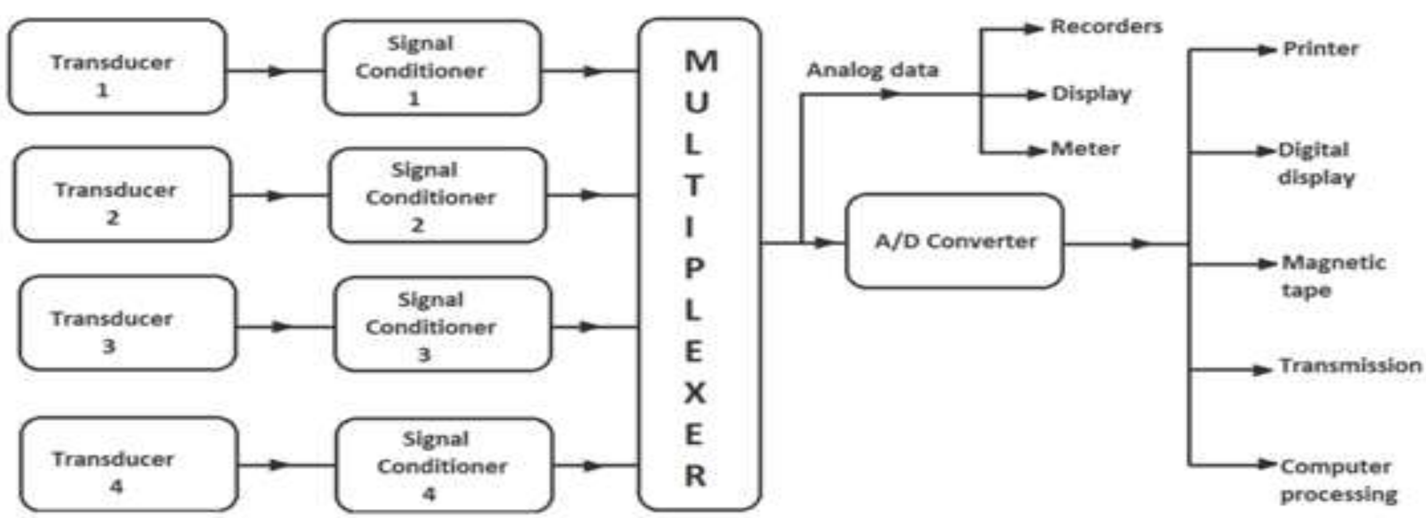
The digital data have more advantages over analog data. Some of those are:

- easy and fast processing,
- easy and fast transmission,
- easy display,
- less storage space is required,
- more accurate.

Due to these advantages, mostly the digital data acquisition system is preferred.

Data Acquisition System Block Diagram

A generalized **data acquisition system block diagram** is shown in Figure.



Generalised Data Acquisition System

The function of each block is as under:

Transducers: They are converting physical quantities (such as temperature, pressure, etc.) into electrical quantities, or measuring electrical quantities directly. They collect data from the physical world.

The most commonly used transducers are:

- RTDs, thermocouples, and thermistors for temperature measurements.
- Photosensors for light measurements.
- Strain gages, piezoelectric transducers for force and pressure measurements.
- Microphone for sound measurements.
- Potentiometer, LVDT, optical encoder for position and displacement measurements.

Signal Conditioning Unit: The signal produced by the transducers may or may not be very suitable for our system to work properly. It may be very weak, very strong or may have some noise.

To convert this signal into the most suitable form, amplification, and filtration is done respectively by signal conditioning unit. So the signal conditioning unit converts electrical signals in the most suitable form.

Multiplexer: The multiplexer receives multiple analog inputs and provides a single output signal according to the requirements. If a separate channel is used for each quantity, the cost of installation, maintenance, and periodic replacement becomes high. Therefore, a single channel is used which is shared by various quantities.

Analog to Digital (A/D) Converters: The data is converted into digital form by A/D converters.

After the conversion of data into digital form, it is displayed with the help of oscilloscopes, numerical displays, panel meters to monitor the complete system.

Also, the data can be either permanently or temporarily stored or recorded according to the requirement. The data is recorded on optical, ultraviolet, stylus or ink recorders for future use.

Objectives of Data Acquisition System

- It must collect the necessary data at the correct speed.
- It must use all the data efficiently to inform the operator about the state of the system.
- It must monitor the complete system operation to maintain on-line optimum and safe operations.
- It must be able to summarize and store data for the diagnosis of operation and record purpose.
- It must be flexible for future requirements.
- It must be reliable and not have a downtime of more than 0.1%.
- It must provide an effective communication system.

Applications of Data Acquisition System

The data acquisition system is used in industrial and scientific fields like aerospace, biomedical and telemetry industries.

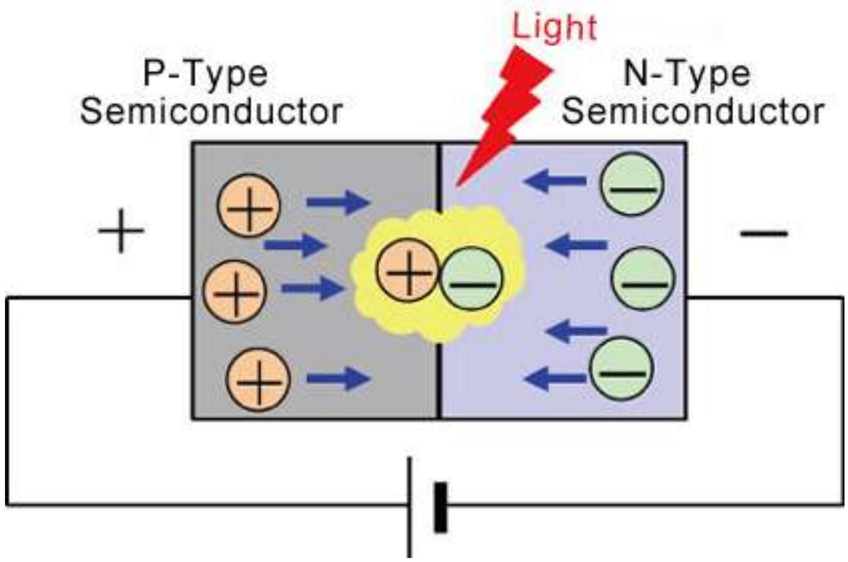
DATA DISPLAY & STORAGE DEVICE

A **display device** is an output device for presentation of information in visual[1] or tactile form (

1. **LIGHT EMITTING DIODE**- The "**Light Emitting Diode**" or LED as it is more commonly called, is basically just a specialised type of diode as they have very similar electrical characteristics to a PN junction diode. This means that an LED will pass current in its forward direction but block the flow of current in the reverse direction.

Light emitting diodes are made from a very thin layer of fairly heavily doped semiconductor material and depending on the semiconductor material used and the amount of doping, when forward biased an LED will emit a coloured light at a particular spectral wavelength.

When the diode is forward biased, electrons from the semiconductors conduction band recombine with holes from the valence band releasing sufficient energy to produce photons which emit a monochromatic (single colour) of light. Because of this thin layer a reasonable number of these photons can leave the junction and radiate away producing a coloured light output.



2. LIQUID CRYSTAL DIODE (LCD)

The LCD is defined as the **diode** that uses **small cells** and the **ionised gases** for the **production of images**. The LCD works on the **modulating property of light.** The light modulation is t**he technique of sending and receiving the signal** through the **light**. The **liquid crystal consumes** a small amount of energy because they are the **reflector and the transmitter of light.** It is normally used for **seven segmental display.**

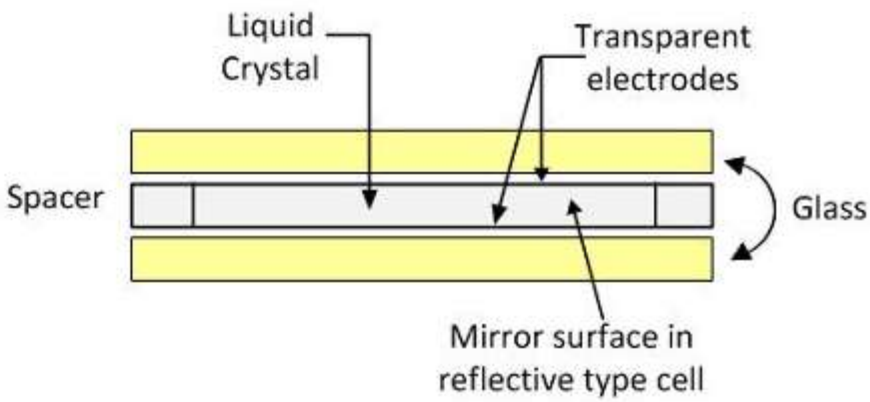
CONSTRUCTION OF LCD

The liquid crystals are the organic compound which is in liquid form and shows the property of optical crystals. The layer of liquid crystals is deposited on the inner surface of glass electrodes for the scattering of light. The liquid crystal cell is of two types; they are Transmittive Type and the Reflective Type.

Transmittive Type – In transmitter cell both the glass sheets are transparent so that the light is scattered in the forward direction when the cell becomes active.

Reflective Type – The reflective type cell consists the reflecting surface of the glass sheet on one end. The light incident on the front surface of the cell is scattered by the activated cell.

Both the reflective and transmittive type cells appear brights, even under small ambient light conditions.

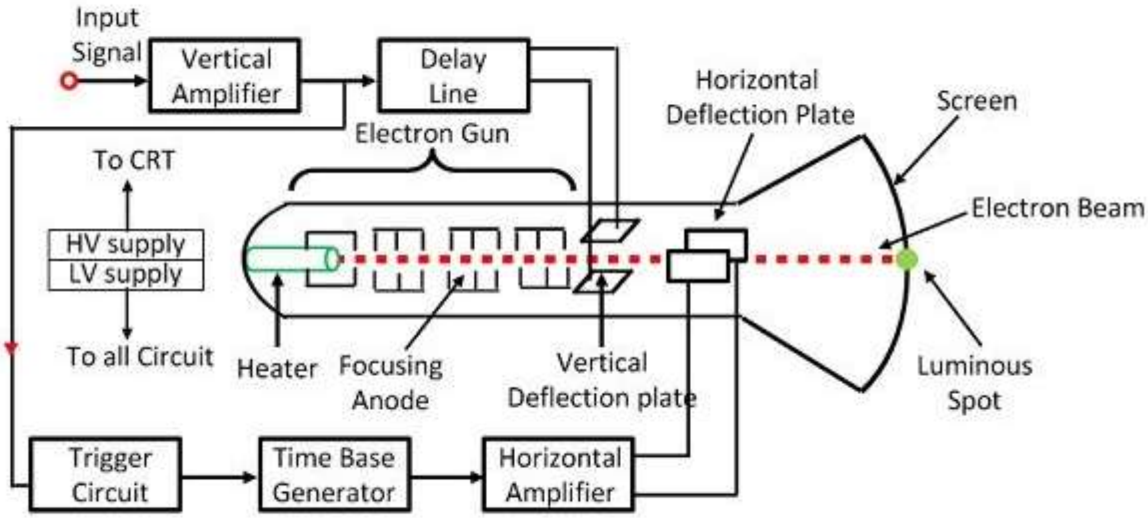


3. CATHODE RAY OSCILLOSCOPE (CRO)

The cathode ray oscilloscope (CRO) is a type of electrical instrument which is used for showing the measurement and analysis of waveforms and others electronic and electrical phenomenon. It is a very fast X-Y plotter shows the input signal versus another signal or versus time. The CROs are used to analyse the waveforms, transient, phenomena, and other time-varying quantities from a very low-frequency range to the radio frequencies.

Working of Cathode Ray Oscilloscope

When the electron is injected through the electron gun, it passes through the control grid. The control grid controls the intensity of electron in the vacuum tube. If the control grid has high negative potential, then it allows only a few electrons to pass through it. Thus, the dim spot is produced on the lightning screen. If the negative potential on the control grid is low, then the bright spot is produced. Hence the intensity of light depends on the negative potential of the control grid.



After moving the control grid the electron beam passing through the focusing and accelerating anodes. The accelerating anodes are at a high positive potential and hence they converge the beam at a point on the screen.

STOARGE:

A **recorder** records electrical and non-electrical quantities as a **function** of time. Currents and voltages can be recorded directly while the non-electrical quantities are recorded indirectly by first converting them to equivalent currents or voltages with the help of sensors or transducers

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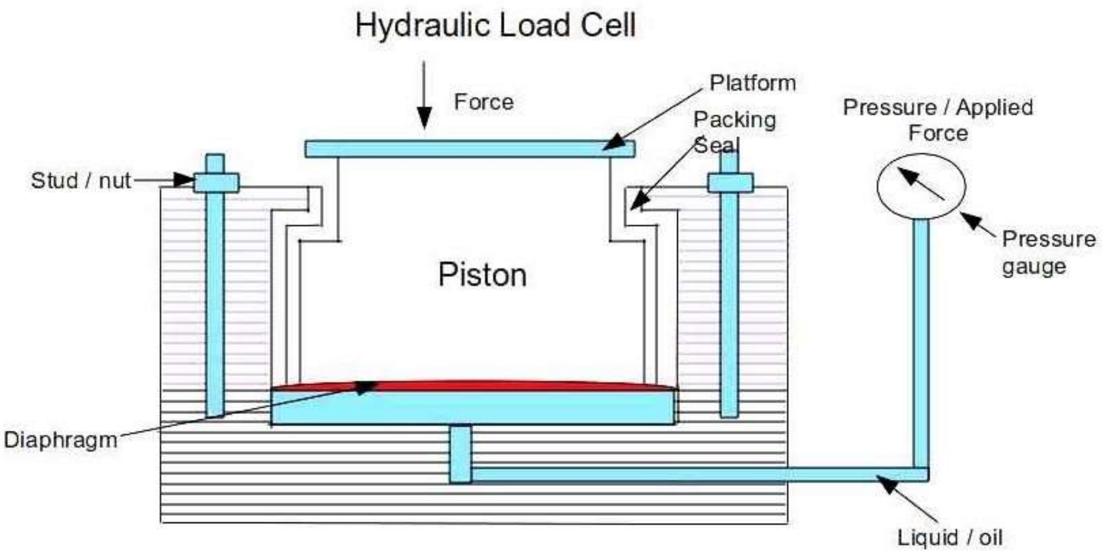
PRINCIPLE OF HYDRAULIC LOAD CELL

When a force is applied on a liquid medium contained in a confined space, the pressure of the liquid increases. This increase in pressure of the liquid is proportional to the applied force. Hence a measure of the increase in pressure of the liquid becomes a measure of the applied force when calibrated.

Hydraulic Load Cell



The main parts of a hydraulic load cell are as follows

- A dirphragm
- A piston with a loading platform (as shown in figure) placed on top of the diaphragm.
- A liquid medium which is under a pre-loaded pressure is on the other side of the diaphragm.
- A pressure gauge (bourdon tube type) connected to the liquid medium.
-

Operation of Hydraulic Load Cell

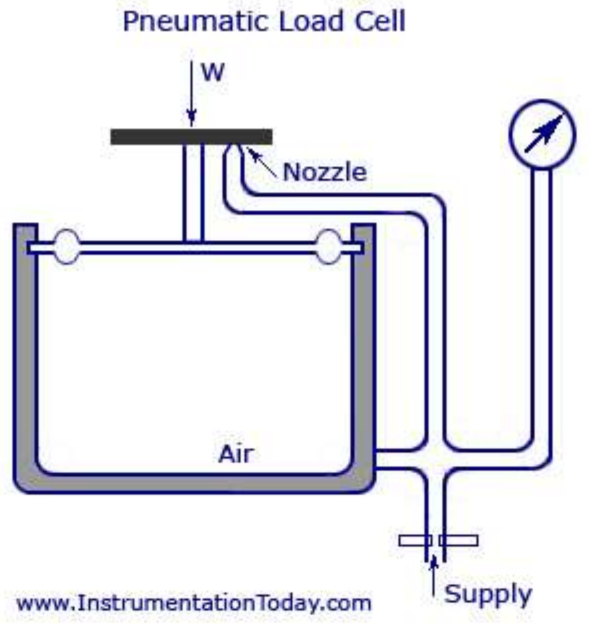
- The force to be measured is applied to the piston.
- The appilied force moves the piston downwards and deflects the diaphragm and this deflection of the diaphragm increases the pressure in the liquid medium (oil).
- This increase in pressure of the liquid medium is proportional to the applied force. The increase in pressure is measured by the pressure gauge which is connected to the liquid medium.
- The pressure is calibrated in force units and hence the indication in the pressure gauge becomes a measure of the force applied on the piston.

Important Points About Hydraulic Load cell:

- As the hydraulic load cell is sensitive to pressure changes, the load cell should be adjusted to zero setting before using it to measure force.
- This hydraulic load cell have an accuracy of the order of 0.1 percent of its scale and can measure loads upto upto 2.5*10^5 Kgf
- The resolution is about 0.02 percent.

PNEUMATIC LOAD CELLS

Pneumatic load cells use air pressure applied to one end of a diaphragm, and it escapes through the nozzle placed at the bottom of the load cell, which has a pressure gauge inside of the cell.



For the pneumatic load cell, in an application of a mass to the cell, it causes the deflection of a diaphragm which acts as a variable restriction in a nozzle-fin mechanism. The output pressure measured in the cell is approximately proportional to the magnitude of the gravitational force on the applied mass.

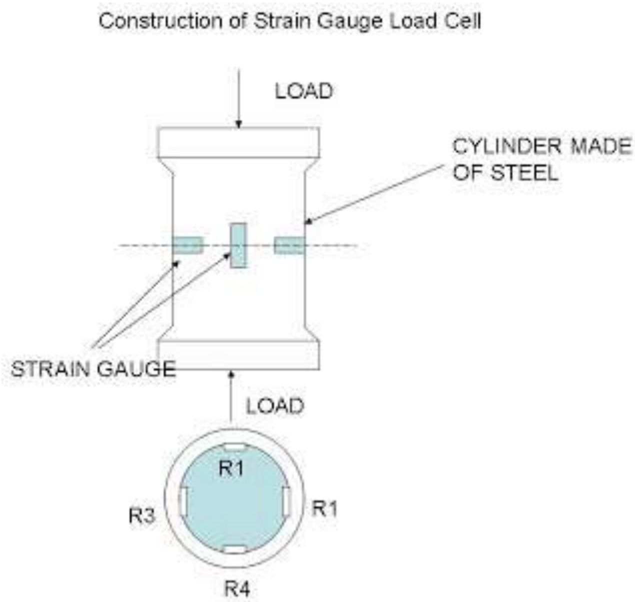
Strain Gauge Load Cell:

A strain gauge load cell is comprised of a solid metal body (or “spring element”) on which strain gauges have been secured. The body is usually made of aluminum, alloy steel, or stainless steel which makes it very sturdy but also minimally elastic. When a load is applied, the body of the load cell is slightly deformed, but, unless overloaded, always returns to its original shape. In response to the body shape changes, the strain gauges also change shape. This, in turn, causes a change in the electrical resistance of the strain gauge which can then be measured as a voltage change. Since this change in output is proportional to the amount of weight applied, the weight of the object can then be determined from the change in voltage.

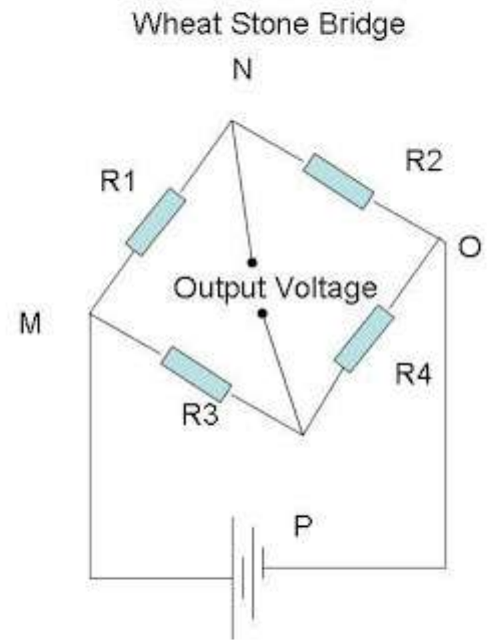
How does a resistive load cell works ?

A load cell is made by using an elastic member (with very highly repeatable deflection pattern) to which a number of strain gauges are attached.

The main parts of the strain gauge load cell are as follows. They are a cylinder made up of steel on which four identical strain gauge are mounted and out of four strain gauges, two of them (R1 and R4) are mounted along the direction of the applied load(vertical gauges). The other two strain gauges (R2 and R3 Horizontal gauges) are mounted circumferentially at right angles to gauges R1 and R4.



Operation of strain gauge Load cell



When there is no load (force) on the steel cylinder, all the four gauges will have the same resistance. As the terminals N and P are at the same potential, the wheat stone bridge is balanced and hence the output voltage will be zero.

Now the load (force) to be measured (say compression force) is applied on the steel cylinder. Due to this, the vertical gauges R1 and R4 will under go compression and hence there will be a decrease in resistance. At the same time, the horizontal gauges R2 and R3 will under go tension and there will be an increase in resistance. Thus when strained, the resistance of the various gauges change.

Now the terminal N and P will be at different potential and the change in output voltage due to the applied load (force) becomes a measure of the applied load force when calibrated.

Strain Gauge Load Cell Working:

The Strain Gauge Load Cell Working is used to weigh extremely heavy loads. A length of bar, usually steel, is used as the active element. The weight of the load applies a particular stress to the bar. The amount of strain which results in the bar for different values of applied stress is determined, so that the strain may be used as a direct measure of the stress causing it.

The load cell shown in Fig. is a good example of the use of strain gauges in weighing operations.

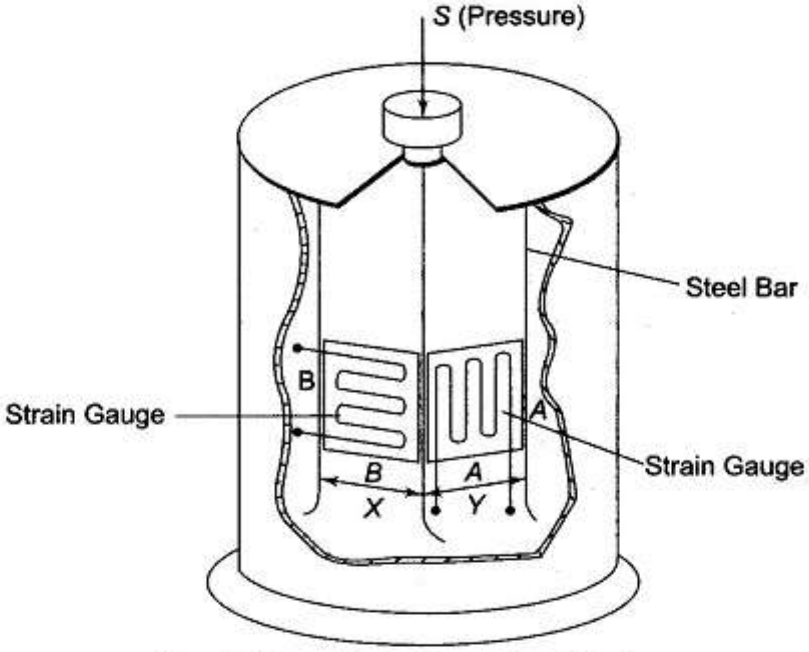


Fig. 13.30 Strain Gauge Load Cell

As the stress is applied along the direction of S (shown by the arrow in Fig.), the steel bar experiences a compression along that axis and an expansion along the X and Y axes. As a result, gauge A experiences a decrease in resistance, while gauge B undergoes an increase in resistance. When these two gauges and the gauges on the two remaining sides of the steel are connected to form a bridge circuit, four times the sensitivity of a simple gauge bridge is obtained. This makes the load cell sensitive to very small values of applied stress, as well as to extremely heavy loads.

Wheatstone Bridge Circuit

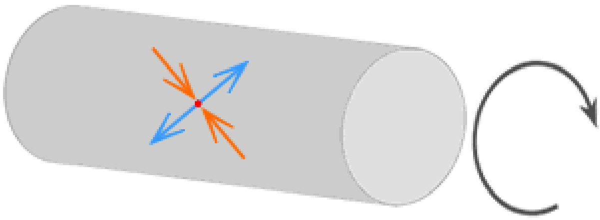
The four strain gauges are configured in a Wheatstone Bridge configuration with four separate resistors connected as shown in what is called a Wheatstone Bridge Network. An excitation voltage - usually 10V is applied to one set of corners and the voltage difference is measured between the other two corners. At equilibrium with no applied load, the voltage output is zero or very close to zero when the four resistors are closely matched in value. That is why it is referred to as a balanced bridge circuit.

When the metallic member to which the strain gauges are attached, is stressed by the application of a force, the resulting strain - leads to a change in resistance in one (or more) of the resistors. This change in resistance results in a change in output voltage. This small change in output voltage (usually about 20 mVolt of total change in response to full load) can be measured and digitized after careful amplification of the small milli-volt level signals to a higher amplitude 0-5V or 0-10V signal.

These load cells have been in use for many decades now, and can provide very accurate readings but require many tedious steps during the manufacturing process

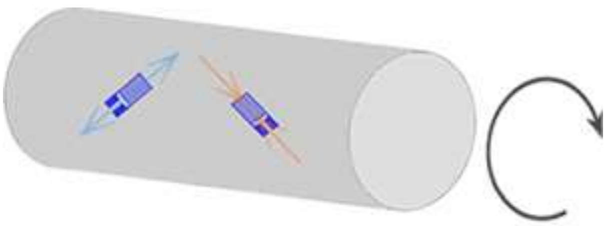
TORQUE SENSOR MEASUREMENT PRINCIPLE

The most common torque sensor measurement principle uses bonded strain gauge technology, where the strain gauges are bonded to a suitably designed shaft. Torque transducers with a circular shaft and with strain gauges applied at 45deg is a design that has been around for many years. However, the design and configuration of the device will be dictated by the application and the shaft may well be solid or hollow and the cross-section might differ with either a cruciform, square or the other custom design, in order to gain the maximum signal output available from the measurement.



Shear Stresses in a Torque Shaft

When torsion is applied to the shaft causing it to twist, shear stresses are induced. These are measured by bonding the strain gauges at 45° to the horizontal torque axis. As the shear stress induced in the shaft is the same throughout its length, the strain gauges can be bonded at any point along its length. However, it is normal practice to place them in the centre, as far as possible from spurious stress that can be induced at the mechanical interfaces.



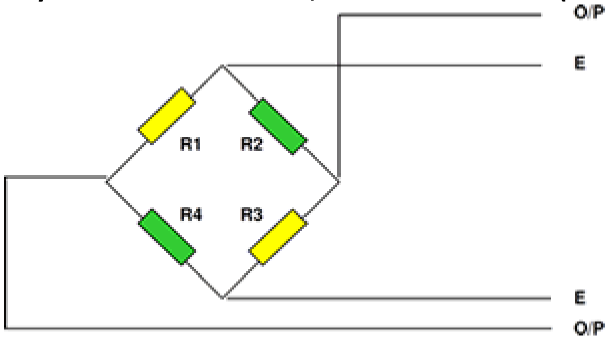
Strain Gauges Bonded to a Torque Shaft

Getting an Electrical Signal from the Torque Transducer

Typically four strain gauges are bonded and connected into a Wheatstone bridge configuration with temperature compensation components included within the bridge circuitry. With an excitation voltage applied to the bridge and torque induced into the shaft, an electrical output linearly proportionate to that torque will result.

For the strain gauged shaft to become a useful measuring instrument (torque sensor) it is necessary to calibrate it with a known reference standard. This can be either weight applied to the end of a cantilever arm of known length, or in line with a reference standard torque transducer.

The completed Wheatstone Bridge requires a stable DC supply to excite the circuit. This is usually 5Vdc or 10Vdc but can be any value from 1Vdc up to 18Vdc.



The Wheatstone Bridge Circuit

Torque Sensors and Torque Sensor Instrumentation

These low-level millivolt signals are compatible with a vast range of bespoke strain gauge sensors such as load cells and pressure transducers and their associated instrumentation. These instruments include digital displays, analogue and digital amplifiers. Typical analogue amplifiers will generate a higher level voltage (0-5Vdc, 0-10Vdc) or current (0-20mA, 4-20mA) for additional processing.

Typically digital amplifiers also provide RS232 or RS485 output using either industry standard or protocols specific to an industry such as MODbus. In many cases the signal conditioning is built in to the device, giving a direct analogue or digital output for further processing.

One of the latest developments to be incorporated into torque sensors in radio telemetry operating in the 2.4GHz frequency band. Operating with very low current consumption enables the use of small batteries on a power source. This latest technology has led to the design of contact-less rotary torque transducers.

A further benefit of radio telemetry torque sensors is their low cost (up to 50%) compared with equivalent slip ring or brushless types.

Selection of an appropriate device will be based on the following amongst other factors, all of which will impact on the accuracy and cost of the measurement, these include:

- Speed of rotation
- The environment in which the measurement is taking place
- The mechanical connection
- Test duration
- Information required of the measurement being taken

Strain gauge torsionmeter

With this device four strain gauges are mounted onto the shaft, as shown in Figure below. The twisting of the shaft as a result of an applied torque results in a change in resistance of the strain gauge system or bridge. Brushes and sliprings are used to take off the electrical connections and complete the circuit, as shown. More recently use has been made of the resistance change converted to a frequency change.

A frequency converter attached to the shaft is used for this purpose: this frequency signal is then transmitted without contact to a digital frequency receiver. When a torque is applied to the shaft, readings of strain and hence torque can be made.

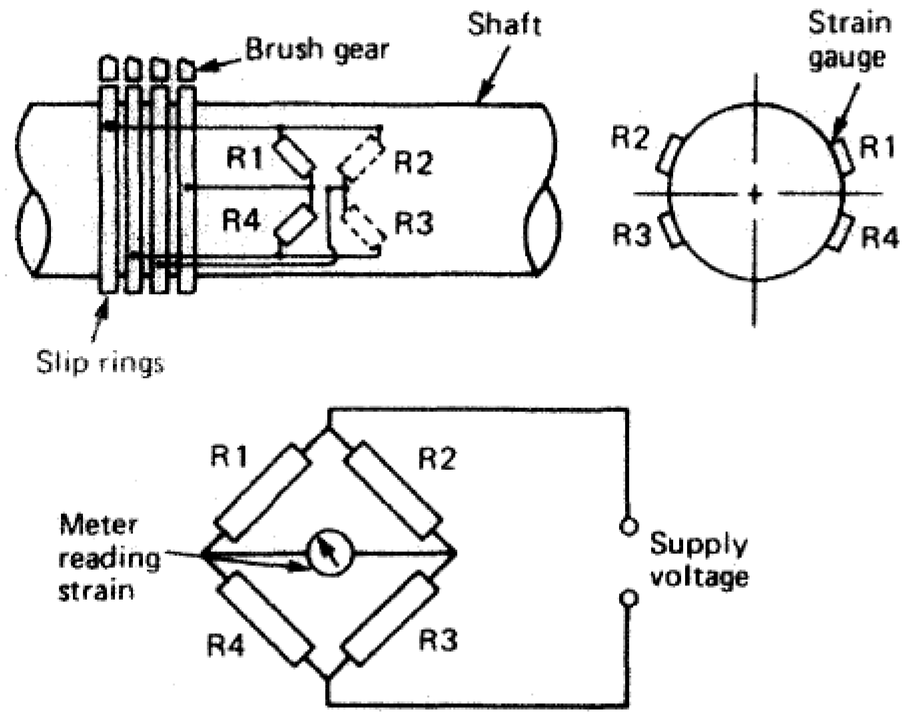


Figure Strain gauge torsionmeter

PRESSURE MEASUREMENTS:

Measurement of Pressure with the Manometer

Pressure is defined as a force per unit area - and the most accurate way to measure low air pressure is to balance a column of liquid of known weight against it and measure the height of the liquid column so balanced. The units of measure commonly used are inches of mercury (in. Hg), using mercury as the fluid and inches of water (in. w.c.), using water or oil as the fluid.

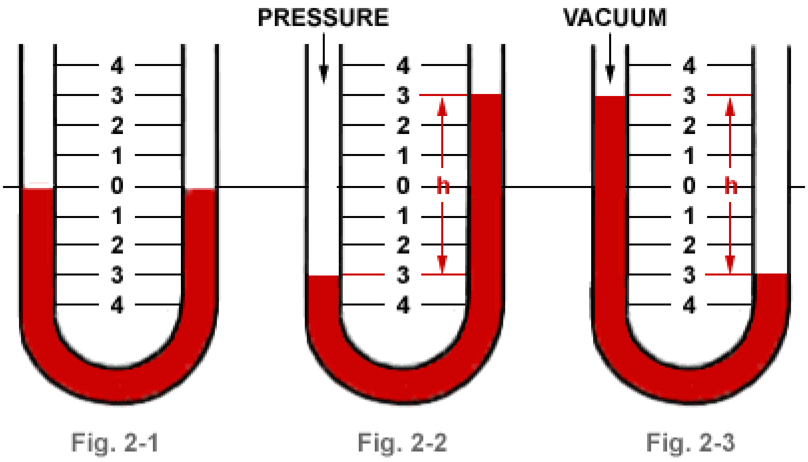


Fig. 2-1. In its simplest form the manometer is a U-tube about half filled with liquid. With both ends of the tube open, the liquid is at the same height in each leg.

Fig. 2-2. When positive pressure is applied to one leg, the liquid is forced down in that leg and up in the other. The difference in height, "h," which is the sum of the readings above and below zero, indicates the pressure.

Fig. 2-3. When a vacuum is applied to one leg, the liquid rises in that leg and falls in the other. The difference in height, "h," which is the sum of the readings above and below zero, indicates the amount of vacuum.

Instruments employing this principle are called manometers. The simplest form is the basic and well-known U-tube manometer. (Fig. 2-1). This device indicates the difference between two pressures (differential pressure), or between a single pressure and atmosphere (gage pressure), when one side is open to atmosphere. If a U-tube is filled to the half way point with water and air pressure is exerted on one of the columns, the fluid will be displaced. Thus one leg of water column will rise and the other falls. The difference in height "h" which is the sum of the readings above and below the half way point, indicates the pressure in inches of water column.

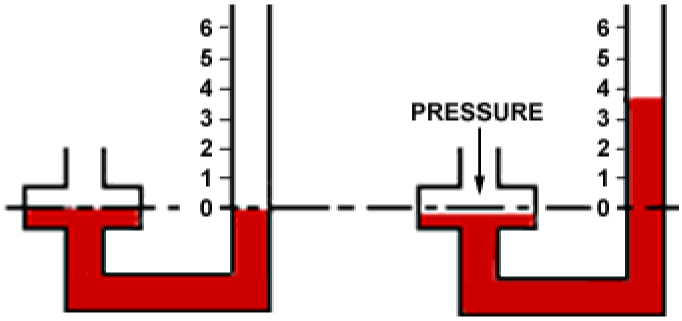


Fig. 2-4

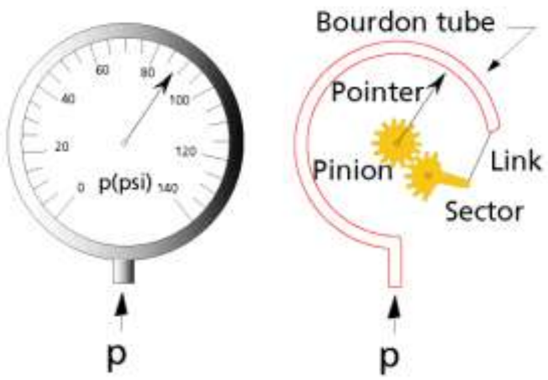
Fig. 2-4. At left, equal pressure is imposed on the fluid in the well and in the indicating tube. Reading is zero. At the right, a positive pressure has been imposed on the liquid in the well causing the level to go down very slightly. Liquid level in indicating tube has risen substantially. Reading is taken directly from scale at liquid level in indicating tube. The scale has been compensated for the drop in level in the well.

The U-tube manometer is a primary standard because the difference in height between the two columns is always a true indication of the pressure regardless of variations in the internal diameter of the tubing. This principle makes even the Dwyer Slack Tube® roll-up manometer as accurate as a laboratory instrument. This provides a real convenience to the person who might otherwise have to board an airplane carrying a 60" long rigid glass U-tube manometer.

MEASUREMENT OF PRESSURE WITH ELASTIC ELEMENTS

Bourdon Tube Pressure Gauge Working

Bourdon tube pressure gauges are extensively used for local indication. This type of pressure gages were first developed by E. Bourdon in 1849. Bourdon tube pressure gauges can be used to measure over a wide range of pressure: form vacuum to pressure as high as few thousand psi. It is basically consisted of a C-shaped hollow tube, whose one end is fixed and connected to the pressure tapping, the other end free, as shown in fig. The cross section of the tube is elliptical.



When pressure is applied, the elliptical tube (Bourdon tube) tries to acquire a circular cross section; as a result, stress is developed and the tube tries to straighten up. Thus the free end of the tube moves up, depending on magnitude of pressure. A deflecting and indicating mechanism is attached to the free end that rotates the pointer and indicates the Pressure reading. The materials used are commonly Phosphor Bronze, Brass and Beryllium Copper. For a 2″ overall diameter of the C-tube the useful travel of the free end is approximately 1/8″ . Though the C-type tubes are most common, other shapes of tubes, such as helical, twisted or spiral tubes are also in use.

VACCUM MEASUREMENTS

MCLEOD GAUGE

McLeod Gauge is a **vacuum gauge** that uses the same principle as that of a **manometer**. By using the pressure dividing technique, its range can be extended from a value of 10^{-4} Torr. The basic principle is called the multiple compression technique. It is shown in the figures below. If there are two bulbs A and B connected with the McLeod and test gauges through capillary tubings, the pressure on the right hand side of the test gauge is very small and the capillary connection between T and bulb B very long, then the flow law can be written as

V.dp2/dt = K.(p1-p2)

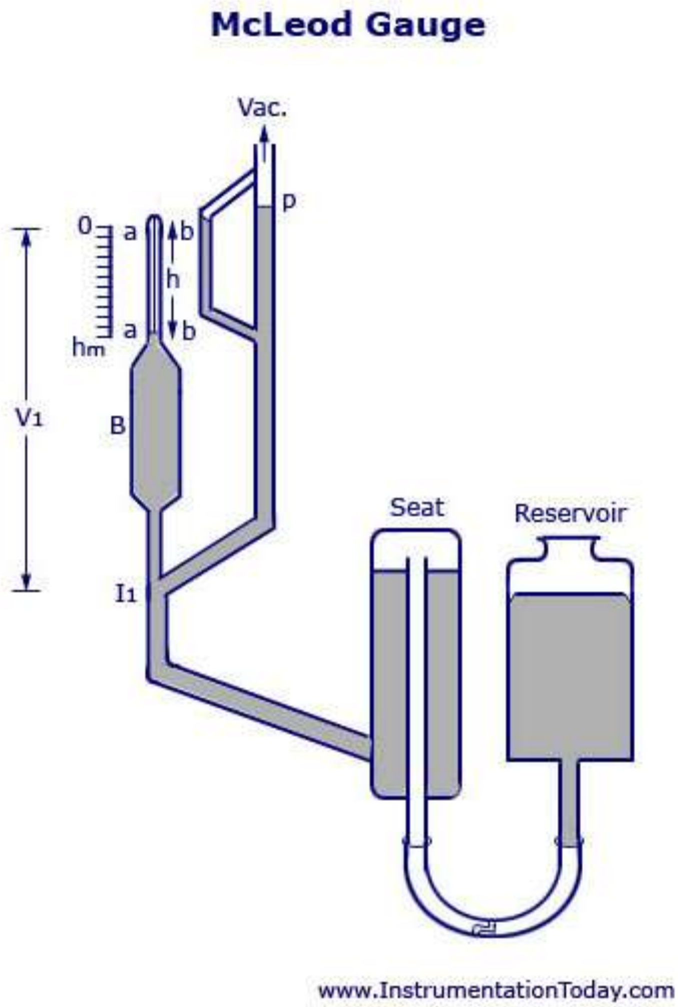
V- Volume of the bulb

dp2/dt – Pressure Gradient in time between the two elements

K – Flow conductance in the capillary.

As p2 is very small when compared to p1, the flow rate remains practically constant and is proportional to the pressure. This forms the basis of the calibration.

There are many variations of the McLeod Gauge. The basic construction is shown in the figure below.

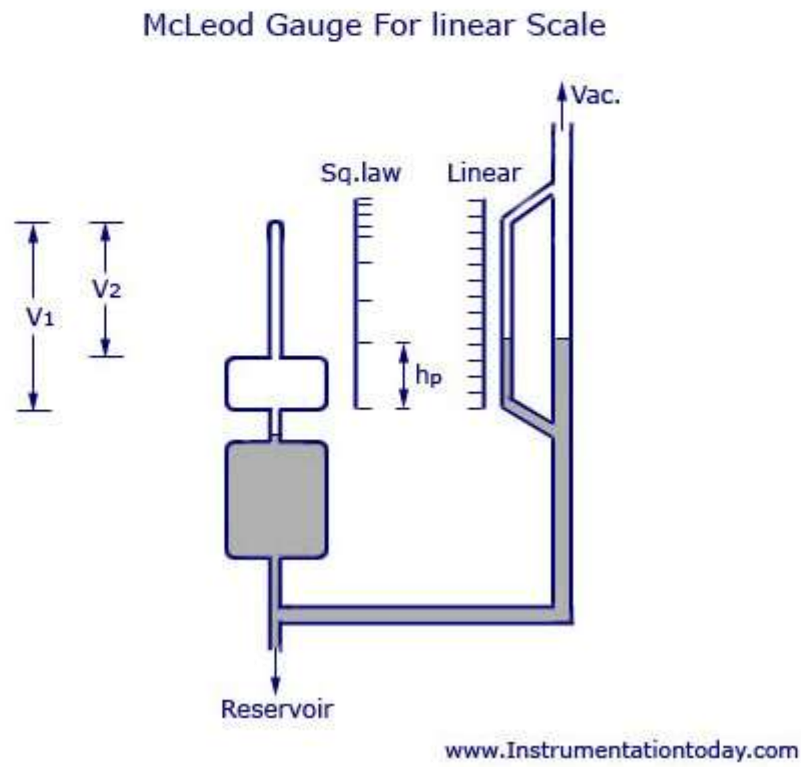


McLeod Gauge

Working

The gauge is used to compress a small quantity of low pressure gas to produce a readable large pressure. Bulb B of the gauge is attached to capillary aa'. The mercury level in the gauge is lowered up to l1 by lowering the reservoir, thereby allowing a little process fluid to enter B. By raising the reservoir, the gas is now compressed in the capillary aa' till mercury rises to the zero mark in the side tube and capillary bb'. The capillary bb' is required to avoid any error due to capillary.

The McLeod gauge is independent of gas composition. If, however, the gas contains condensable material and during compression it condenses, the reading of the gauge is faulty. The gauge is not capable of continuous reading and the scale is of square law type. For linearizing the scale at comparatively higher pressures, a second volume is introduced as shown in the figure below, where the scale shown is linear.



Applications of McLeod gauge

McLeod gauge is used mainly for calibrating other inferential type of gauges. The shortcomings of the McLeod gauge are its fragility and the inability to measure continuously. The vapor pressure of Mercury sets the lower limit of measurement range of the gauge.

Advantages of the McLeod Gauge:

- It is independent of the gas composition.
- It serves as a reference standard to calibrate other low pressure gauges.
- A linear relationship exists between the applied pressure and h
- There is no need to apply corrections to the McLeod Gauge readings.

Limitations of McLeod Gauge:

- The gas whose pressure is to be measured should obey the Boyle's law
- Moisture traps must be provided to avoid any considerable vapor into the gauge.
- It measure only on a sampling basis.
- It cannot give a continuous output.

PIRANI GAUGE

Pirani gauge – A Thermal conductivity Gauge

The Pirani Gauge is a type of Thermal Conductivity Gauges.

Basic principle of Pirani gauge

A conducting wire gets heated when electric current flows through it. The rate at which heat is dissipated from this wire depends on the conductivity of the surrounding media.

The conductivity of the surrounding media in-turn depends on the density of the surrounding media (that is, lower pressure of the surrounding media, lower will be its density). If the density of the surrounding media is low, its conductivity also will be low causing the wire to become hotter for a given current flow, and vice versa.

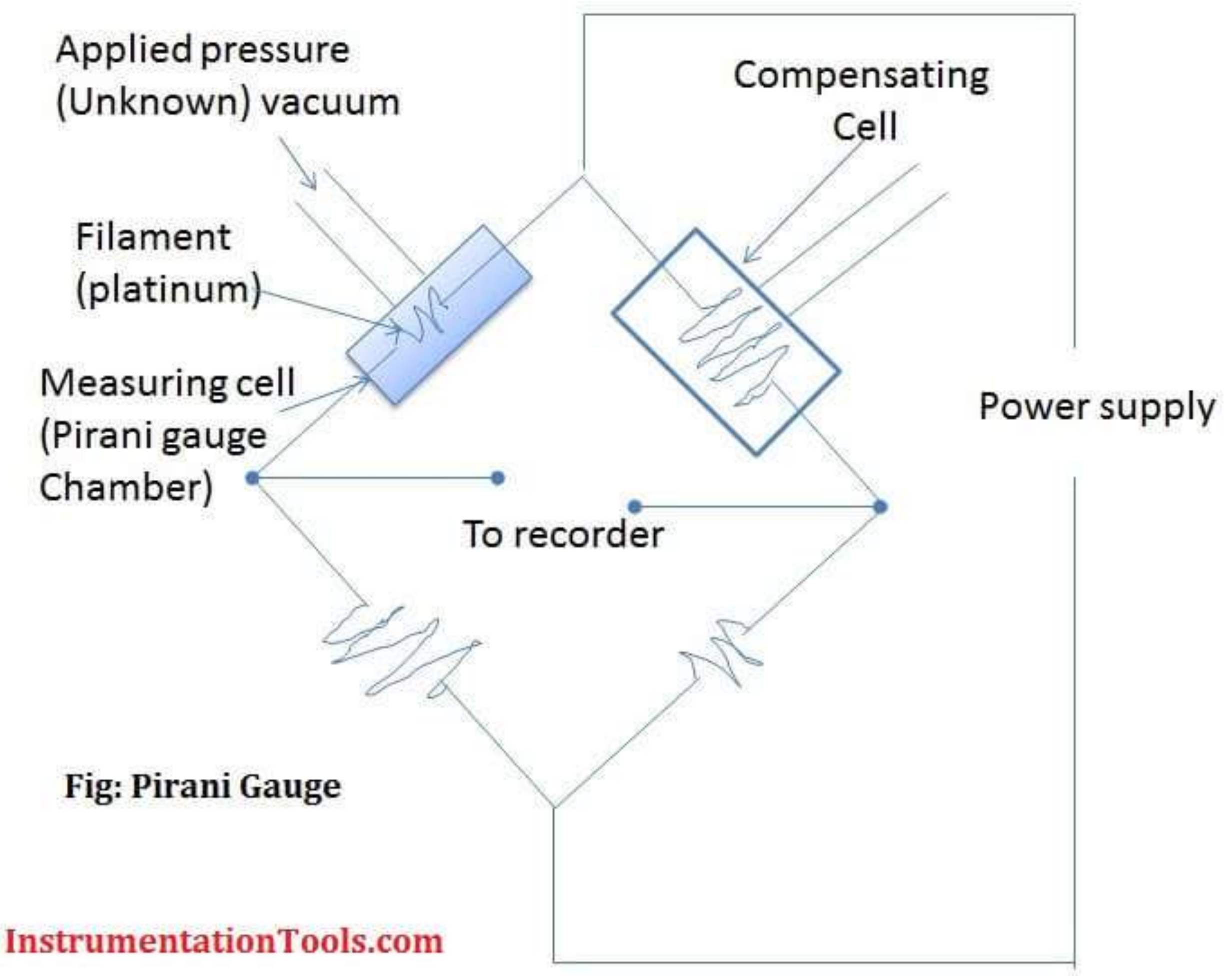


Fig: Pirani Gauge

The main parts of the arrangement are:

1. A pirani gauge chamber which encloses a platinum filament.
2. A compensating cell to minimize variation caused due to ambient temperature changes.
3. The pirani gauge chamber and the compensating cell is housed on a wheat stone bridge circuit as shown in diagram.

Operation of Pirani gauge

1. A constant current is passed through the filament in the pirani gauge chamber. Due to this current, the filament gets heated and assumes a resistance which is measured using the bridge.
2. Now the pressure to be measured (applied pressure) is connected to the pirani gauge chamber. Due to the applied pressure the density of the surrounding of the pirani gauge filament changes. Due to this change in density of the surrounding of the filament its conductivity changes causing the temperature of the filament to change.
3. When the temperature of the filament changes, the resistance of the filament also changes.
4. Now the change in resistance of the filament is determined using the bridge.
5. This change in resistance of the pirani gauge filament becomes a measure of the applied pressure when calibrated.

Applications of Pirani gauge

Used to measure low vacuum and ultra high vacuum pressures.

Advantages of Pirani gauge

1. They are rugged and inexpensive
2. Give accurate results
3. Good response to pressure changes.
4. Relation between pressure and resistance is linear for the range of use.
5. Readings can be taken from a distance.

Limitations of Pirani gauge

1. Pirani gauge must be checked frequently.
2. Pirani gauge must be calibrated from different gases.
3. Electric power is a must for its operation.

IOINIZAION VACCUM GAUGE:

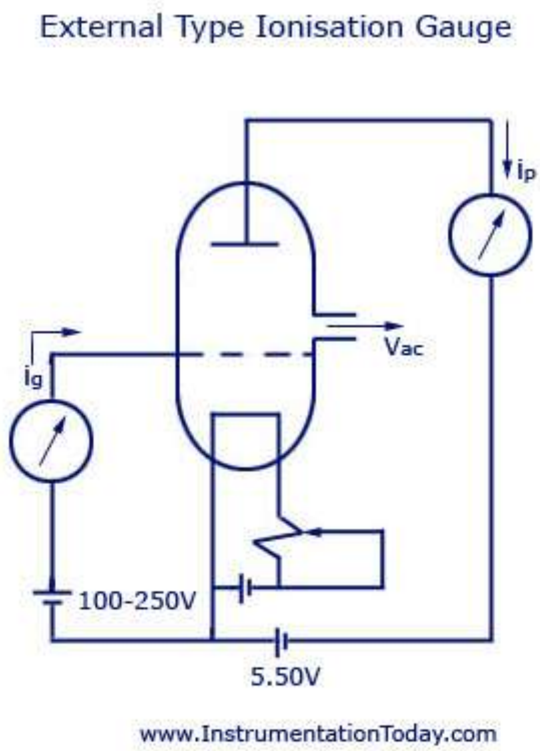
Ionization Gauge is a device that is used to **measure vacuum**. In the hot cathode type, a column of gas is introduced into which, a potential difference V is applied with free electron in the space. This causes the electron with a charge e to acquire a kinetic energy Ve. If the pressure range of the gas in the column goes below a certain limit, called the critical pressure, then corresponding to a voltage larger than the critical voltage Vc, the energy Ve may be high enough to initiate ionization, and positive ions will be produced when the electrons collide with the gas molecules.

The value of Vc is smallest for cesium (3.88V) and largest for helium (24.58V), among monoatomic gases or vapours. For diatomic gases like N2, H2 and so on, it is roughly about 15V. This is known as the ionization potential and at this potential the pressure is also important.

At very low pressures, during the intervals of time for transit from the cathode to the plate in a vacuum chamber, more than one collision is unlikely for an electron. Then for a fixed accelerating potential V>Vc, the number of positive ions formed would vary linearly with the value of pressure. Thus, a determination of the rate of production of positive ions for a given electron current should give a measure of the pressure.

Working

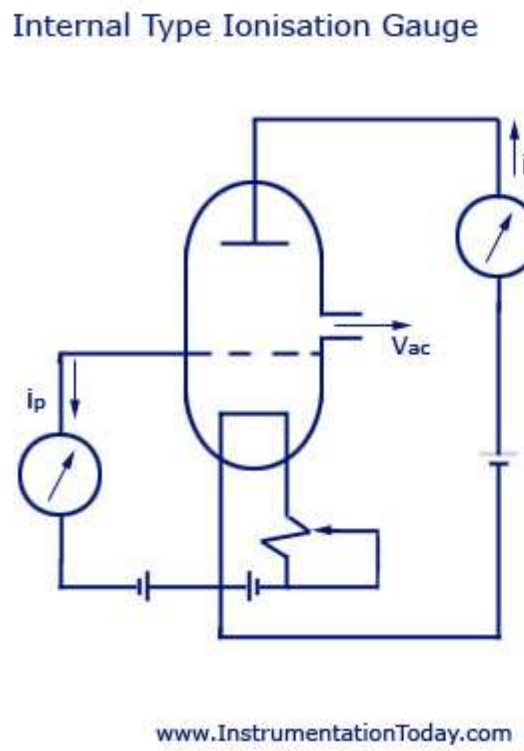
The construction of a hot cathode type ionization gauge consists of a basic vacuum triode. The figure of an external control type hot cathode gauge is shown below.



External Type Ionisation Gauge

The grid is maintained at a large positive potential with respect to the cathode and the plate. The plate is at a negative potential with respect to the cathode. This method is also known as the external control type ionization gauge as the positive ion collector is external to the electron collector grid with reference to the cathode. The positive ions available between the grid and the cathode will be drawn by the cathode, and those between the grid and the plate will be collected by the plate.

The internal control type is shown below. Here the grid is the positive ion collector and the plate is the electron collector.



Internal Type Ionisation Gauge

One of the most popularly used hot filament gauges for industrial applications is the Bayard – Alpert type filament gauge. It consists of a helical grid with a potential of +150 volts. This huge potential attracts the electrons and thus causes gas ionization. At -30 volts, the gas ions are attracted to the central ion collector, thus producing an ion current of 100mA/Torr. This value is then fed to the electronic systems to be amplified and displayed.

The hot cathode ionization gauge is useful in measuring the total pressure of all the gases present in the system. The biggest advantage of this device is its very small response time. This is because of the devices small inertia. The device is used for pressure measurement between the ranges of 10^{-8} to 10^{-3} Torr with an output current varying between 10^{-9} to 10^{-4} A. But this range depends on the gas, other things remaining constant.

Where the pressure is higher than 10^{-3} Torr, the positive ions make a greater impact on the cathode to heat it up and ultimately destroy it. At pressure ranges below 10^{-8} Torr, in external control type, the electrons impact over the grid and radiates soft x-rays, which results in the production of electrons from the plates as secondary emission. These electrons produced will be of the same order as that of the positive ion current in the plate circuit and thus neutralizes this current. Thus the internal control type is known to be a better option to measure pressure as low as 10^{-9} Torr.

When the cathode remains at very temperatures (say 3000 deg C), the gaseous matters present inside may reset with the filament or with themselves particularly at different pressure stages. This may causes the device to produce wrong outputs and may also affect the cathode life. During extreme conditions of high temperatures and low pressures, the presence of any gases inside the device, will be forcefully released, thus causing the pressure to increase. Thus, the electrodes have to be properly treated before use. This can be done only by passing high currents through the electrodes, especially the filament and the grid and by high frequency heating of the plate. To overcome these problems, the **cold cathode type ionization gauge** is also used by many.

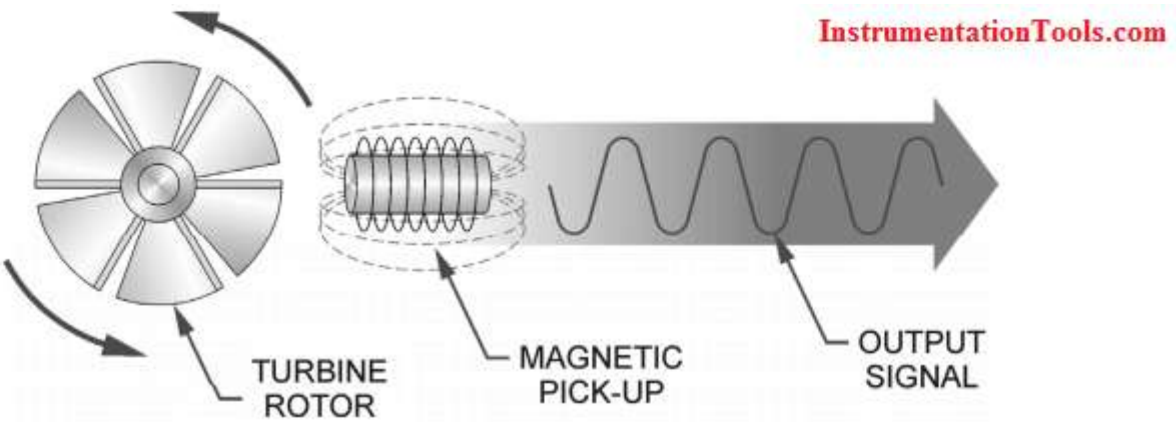
TYPES OF FLOW METER:

1. Turbine Flow Meter

Turbine Flow Meter is a volumetric measuring turbine type. The flowing fluid engages the rotor causing it to rotate at an angular velocity proportional to the fluid flow rate.
The angular velocity of the rotor results in the generation of an electrical signal (AC sine wave type) in the pickup. The summation of the pulsing electrical signal is related directly to total flow.

The frequency of the signal relates directly to flow rate. The vaned rotor is the only moving part of the flow meter.

Turbine Flow Meter



The Turbine flow meter (axial turbine) was invented by **Reinhard Woltman** and is an accurate and reliable flow meter for liquids and gases. It consists of a flow tube with end connections and a magnetic multi bladed free spinning rotor (impeller) mounted inside; in line with the flow. The rotor is supported by a shaft that rests on internally mounted supports.

The Supports in Process Automatics Turbine Flow Meters are designed to also act as flow straighteners, stabilizing the flow and minimizing negative effects of turbulence. The Supports also house the unique open bearings; allowing for the measured media to lubricate the bushes – prolonging the flow meters life span. The Supports are fastened by locking rings (circlips) on each end.

The rotor sits on a shaft ,which in turn is suspended in the flow by the two supports. As the media flows, a force is applied on the rotor wings. The angle and shape of the wings transform the horizontal force to a perpendicular force, creating rotation. Therefore, the rotation of the rotor is proportional to the applied force of the flow.

Because of this, the rotor will immediately rotate as soon as the media induces a forward force. As the rotor cannot turn thru the media on its own, it will stop as soon as the media stops. This ensures an extremely fast response time, making the Turbine Flow Meter ideal for batching applications.

A pick-up sensor is mounted above the rotor. When the magnetic blades pass by the pickup sensor, a signal is generated for each passing blade. This provides a pulsed signal proportional to the speed of the rotor and represents pulses per volumetric unit.; and as such the flow rate too.

Advantages & Disadvantages

- The cost is moderate.
- Very good at clean,
- low viscosity fluids of moderate velocity and a steady rate.
- Turndown is very good as it can read very low compared to the maximum flow.
- They are reliable if put in a clean fluid especially if it has some lubricity.
- AGA and API approved for custody transfers.
- They do cause some pressure drop where that may be a factor such as gravity flows.
- Not reliable for steam
- Bearings wear out.

Applications

In order of magnitude from largest to smallest,

- these are used in oil and gas,
- water and waste water,
- gas utility,
- chemical,
- power, food and beverage,
- aerospace, pharmaceutical,
- metals and mining, and pulp and paper.

2. ELECTROMAGNETIC FLOWMETERS

Working Principle :

Electromagnetic Flowmeters are based on Faraday's Law of Electromagnetic Induction.

In an Electromagnetic Flowmeter, the magnetic field is generated by a set of coils. As the conductive liquid passes through the electromagnetic field, an electric voltage is induced in the liquid which is directly proportional to its velocity. This induced voltage is perpendicular to both, the liquid flow direction and the electromagnetic field direction. The voltage sensed by the electrodes is further processed by the transmitter to give standardised output signal or displayed in appropriate engineering unit.

The flux density of the electromagnetic field in a given Flowmeter and the distance between the electrodes are constant. Therefore, the induced voltage is only a function of liquid velocity.

$$\mathbf{E} = \mathbf{K} \times \mathbf{B} \times \overline{\mathbf{v}} \times \mathbf{D}$$

where **E** : Induced voltage
K : Flow tube constant
B : Magnetic field strength
$\overline{\mathbf{v}}$: Mean flow velocity
and **D** : Electrode spacing

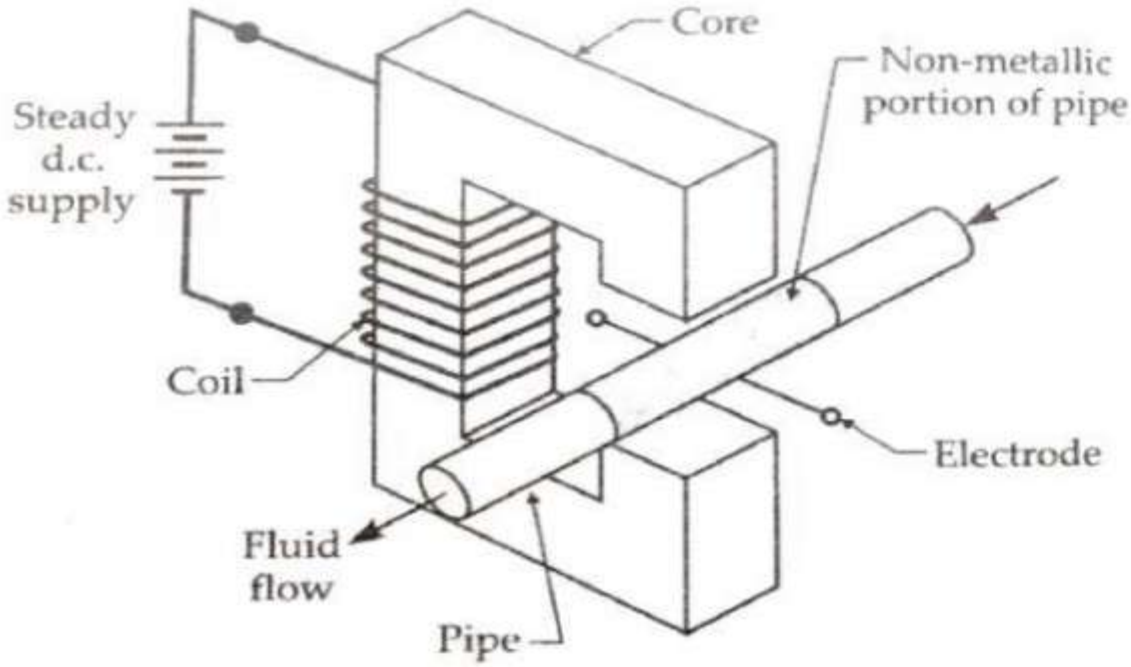
Volume flow is calculated by the equation

$$\mathbf{Q} = \overline{\mathbf{v}} \times \mathbf{D}^2 \times \pi/4$$

Therefore, $\mathbf{Q} = \dfrac{\mathbf{E} \times \mathbf{D} \times \pi}{\mathbf{K} \times \mathbf{B} \times 4}$

The induced voltage is not affected by the physical properties of liquids like temperature, viscosity, pressure, density and conductivity as long as the conductivity of the measured liquid is above the minimum threshold level. For reliable measurement, the pipe must be completely full of liquid.

The electromagnetic field coil assembly is excited by pulsed DC technique which eliminates the interfering noise and provides automatic zero correction.



Advantages:

(1) The obstruction to the flow is almost nil and therefore this type of meters can be used for measuring heavy suspensions, including mud, sewage and wood pulp.

(2) There is no pressure head loss in this type of flow meter other than that of the length of straight pipe which the meter occupies.

(3) They are not very much affected by upstream flow disturbances.

(4) They are practically unaffected by variation in density, viscosity, pressure and temperature.

(5) Electric power requirements can be low (15 or 20 W), particularly with pulsed DC types.

(6) These meters can be used as bidirectional meters.

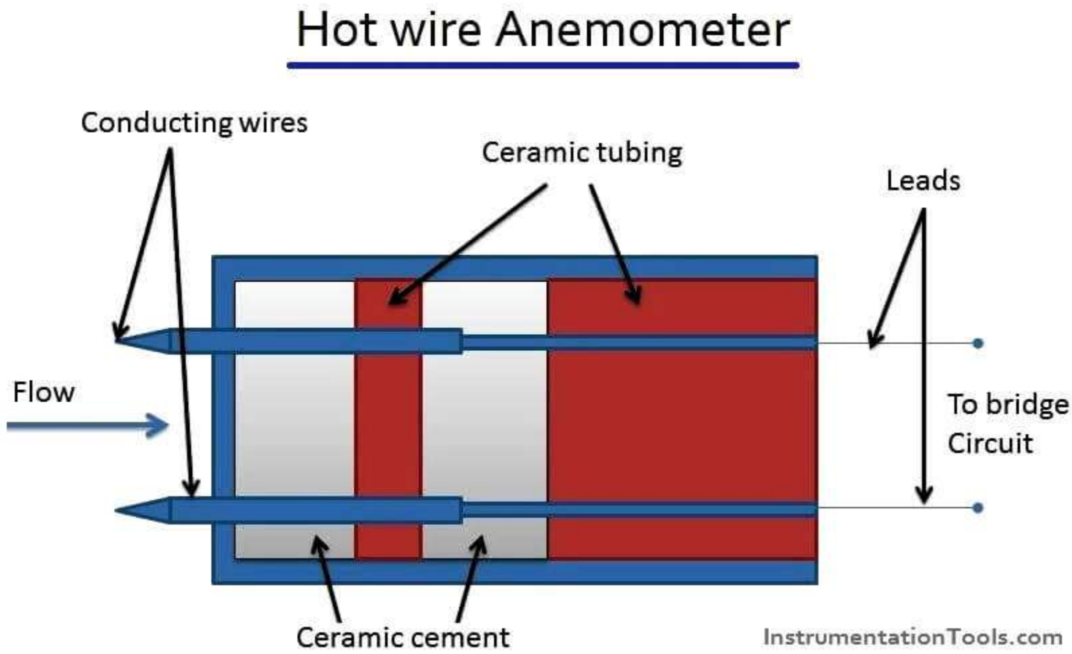
(7) The meters are suitable for most acids, bases, water and aqueous solutions because the lining materials selected are not only good electrical insulators but also are corrosion resistant.

(8) The meters are widely used for slurry services not only because they are obstruction less but also because some of the liners such as polyurethane, neoprene and rubber have good abrasion or erosion resistance.

(9) They are capable of handling extremely low flows.

3. HOT WIRE ANEMOMETER

works When an electrically heated wire is placed in a flowing gas stream, heat is transferred from the wire to the gas and hence the temperature of the wire reduces, and due to this, the resistance of the wire also changes. This change in resistance of the wire becomes a measure of flow rate.



Hot Wire Anemometer Principle

The main parts of the arrangement are as follows:

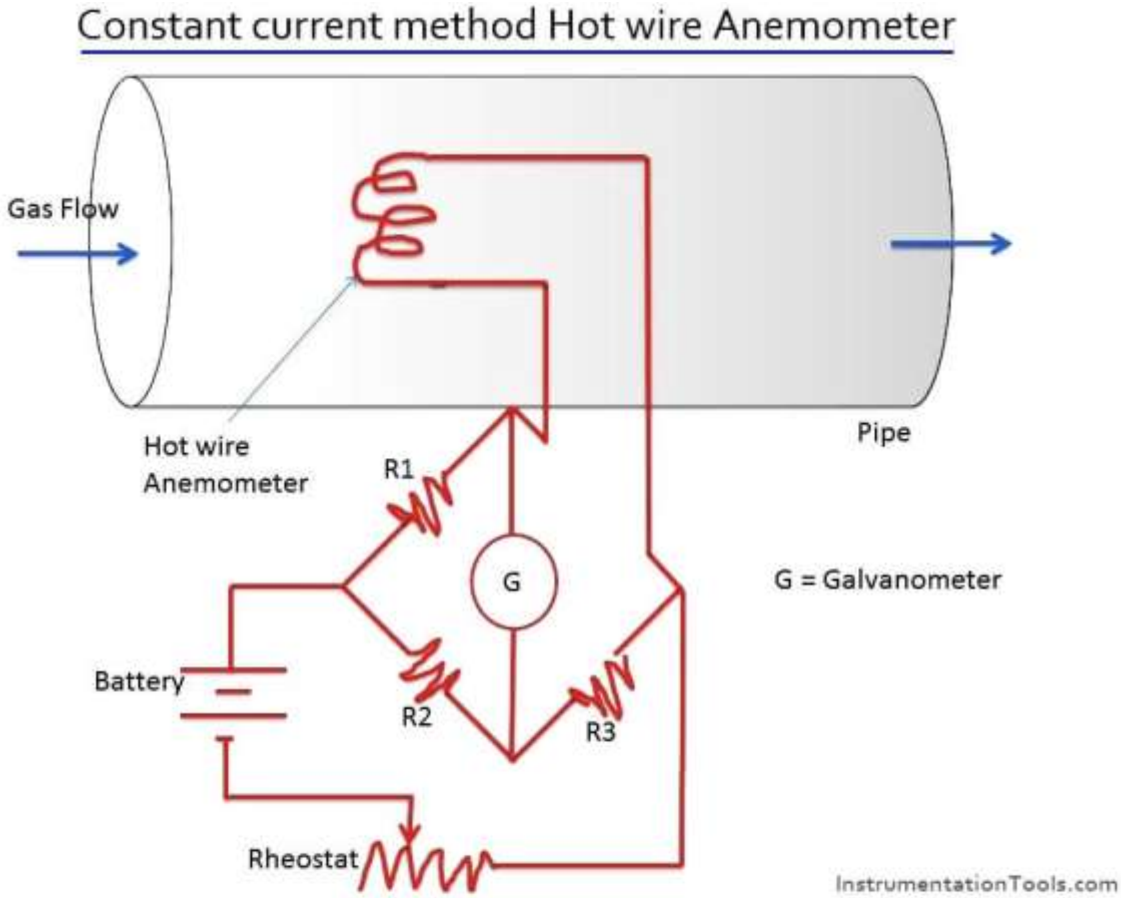
- Conducting wires placed in a ceramic body.
- Leads are taken from the conducting wires and they are connected to one of the limbs of the wheat stone bridge to enable the measurement of change in resistance of the wire.

Types of Hot wire Anemometer

There are two methods of measuring flow rate using a anemometer bridge combination namely:

- Constant current method
- Constant temperature method

Constant current method Hot wire Anemometer



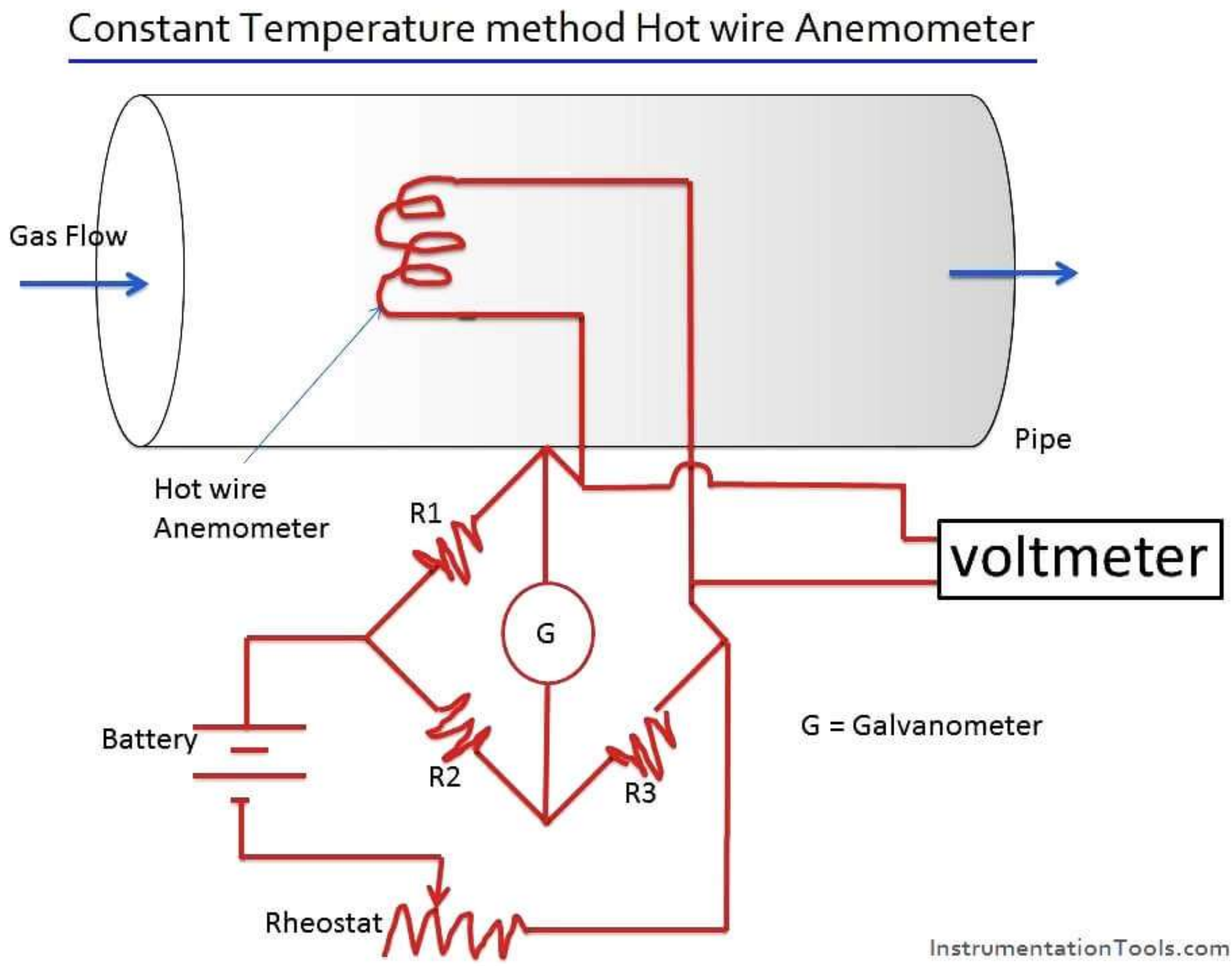
The bridge arrangement along with the anemometer has been shown in diagram. The anemometer is kept in the flowing gas stream to measure flow rate.

A constant current is passed through the sensing wire. That is, the voltage across the bridge circuit is kept constant, that is, not varied.

Due to the gas flow, heat transfer takes place from the sensing wire to the flowing gas and hence the temperature of the sensing wire reduces causing a change in the resistance of the sensing wire. (this change in resistance becomes a measure of flow rate).

Due to this, the galvanometer which was initially at zero position deflects and this deflection of the galvanometer becomes a measure of flow rate of the gas when calibrated.

Constant temperature method Hot wire Anemometer



The bridge arrangement along with the anemometer has been shown in diagram. The anemometer is kept in the flowing gas stream to measure flow rate.

A current is initially passed through the wire.

Due to the gas flow, heat transfer takes place from the sensing wire to the flowing gas and this tends to change the temperature and hence the resistance of the wire.

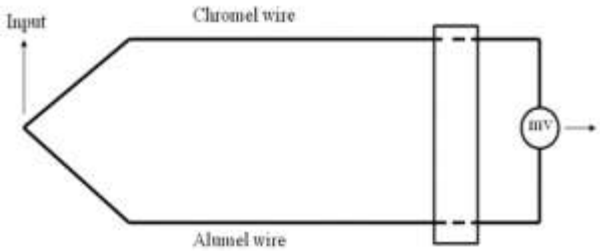
The principle in this method is to maintain the temperature and resistance of the sensing wire at a constant level. Therefore, the current through the sensing wire is increased to bring the sensing wire to have its initial resistance and temperature.

The electrical current required in bringing back the resistance and hence the temperature of the wire to its initial condition becomes a measure of flow rate of the gas when calibrated.

TEMPERATURE MEASUREMENTS METHODS:

1. THERMOCOUPLE

The thermocouple can be defined as a kind of temperature sensor that is used to measure the temperature at one specific point in the form of the EMF or an electric current. This sensor comprises two dissimilar metal wires that are connected together at one junction. The temperature can be measured at this junction, and the change in temperature of the metal wire stimulates the voltages.



Thermocouple

The amount of EMF generated in the thermocouple is very minute (millivolts), so very sensitive devices must be utilized for calculating the e.m.f produced in the circuit. The common devices used to calculate the e.m.f are voltage balancing potentiometer and the ordinary galvanometer. From these two, a balancing potentiometer is utilized physically or mechanically.

Thermocouple Working Principle

The **thermocouple principle** mainly depends on the three effects namely Seebeck, Peltier and Thompson.

See beck-effect

This type of effect occurs among two dissimilar metals. When the heat offers to any one of the metal wire, then the flow of electrons supplies from hot metal wire to cold metal wire. Therefore, direct current stimulates in the circuit.

Peltier-effect

This Peltier effect is opposite to the Seebeck effect. This effect states that the difference of the temperature can be formed among any two dissimilar conductors by applying the potential variation among them.

Thompson-effect

This effect states that as two disparate metals fix together & if they form two joints then the voltage induces the total conductor's length due to the gradient of temperature. This is a physical word that demonstrates the change in rate and direction of temperature at an exact position.

CONSTRUCTION OF THERMOCOUPLE

The construction of the thermocouple is shown below. It comprises of two different metal wires and that are connected together at the junction end. The junction thinks as the measuring end. The end of the junction is classified into three type's namely ungrounded, grounded and exposed junction.

Ungrounded-Junction

In this type of junction, the conductors are totally separated from the protecting cover. The applications of this junction mainly include high-pressure application works. The main benefit of using this function is to decrease the stray magnetic field effect.

Grounded-Junction

In this type of junction, the metal wires as well as protecting cover are connected together. This function is used to measure the temperature in the acidic atmosphere, and it supplies resistance to the noise.

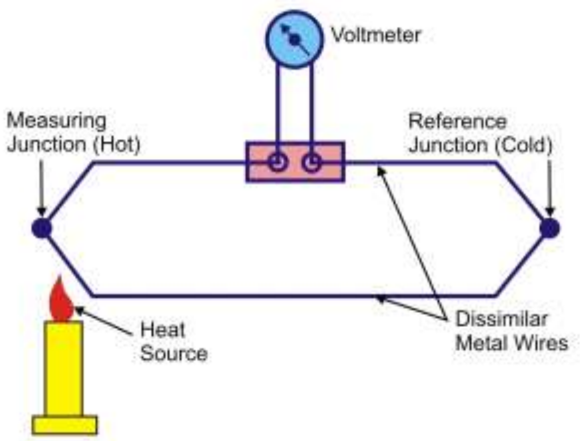
Exposed-Junction

The exposed junction is applicable in the areas where a quick response is required. This type of junction is used to measure the gas temperature. The metal used to make the thermocouple basically depends on the calculating range of temperature.

WORKING OF THERMOCOUPLE

The thermocouple schematic diagram is shown in the below figure. This circuit can be built with two different metals, and that are coupled together by generating two junctions. The two metals are surrounded by the connection through welding.

In the above diagram, the junctions are denoted by P & Q, and the temperatures are denoted by T1, & T2. When the temperature of the junction is dissimilar from each other, then the electromagnetic force generates in the circuit.



Working of Thermocouple

If the temperate at the junction end turn into equivalent, then the equivalent, as well as reverse electromagnetic force, produces in the circuit, and there is no flow of current through it. Similarly, the temperature at the junction end becomes imbalanced, then the potential variation induces in this circuit.

The magnitude of the electromagnetic force induces in the circuit relies on the sorts of material utilized for thermocouple making. The entire flow of current throughout the circuit is calculated by the measuring tools.

The electromagnetic force induced in the circuit is calculated by the following equation

$$\mathbf{E = a\,(\Delta\Theta) + b\,(\Delta\Theta)2}$$

Where $\Delta\Theta$ is the temperature difference among the hot thermocouple junction end as well as the reference thermocouple junction end, a & b are constants

Advantages & Disadvantages of Thermocouple

The advantages include the following.

- Accuracy is high
- It is Robust and can be used in environments like harsh as well as high vibration.
- The thermal reaction is fast
- The operating range of the temperature is wide.
- Wide operating temperature range
- Cost is low and extremely consistent

The disadvantages include the following.

- Nonlinearity
- Least stability
- Low voltage
- Reference is required
- least sensitivity
- The thermocouple recalibration is hard

2. RESISTANCE TEMPERATURE DETECTOR: (RTD)

Resistance Temperature Detectors (RTD) Working Principle
Thermo-resistive Temperature Measuring Devices
A change in temperature causes the electrical resistance of a material to change. The resistance change is measured to infer the temperature change.

There are two types of thermo-resistive measuring devices:

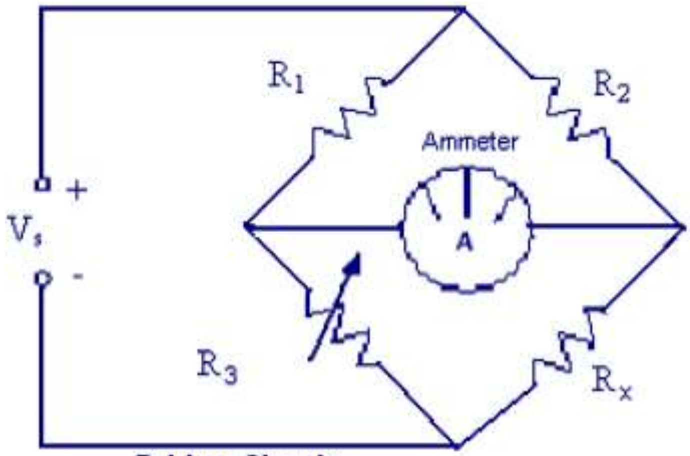
- Resistance temperature detectors (RTD) and
- Thermistors

Resistance Temperature Detectors
A resistance temperature detector (abbreviated RTD) is basically either a long, small diameter metal wire wound in a coil or an etched grid on a substrate, much like a strain gage. Platinum is the most common metal used for RTDs.

Principle of Operation
Resistance Temperature Detectors (RTD) operates on the principle that the electrical resistance of a metal changes predictably in an essentially linear and repeatable manner with changes in temperature. RTD have a positive temperature coefficient (resistance increases with temperature). The resistance of the element at a base temperature is proportional to the length of the element and the inverse of the cross sectional area.

A typical electrical circuit designed to measure temperature with RTDs actually measures a change in resistance of the RTD, which is then used to calculate a change in temperature. The resistance of an RTD increases with increasing temperature, just as the resistance of a strain gage increases with increasing strain.

Bridge Circuit Construction
Figure below shows a basic bridge circuit which consists of three known resistances, R1, R2, and R3 (variable), an unknown variable resistor RX (RTD), a source of voltage, and a sensitive ammeter.



Bridge Circuit

Resistors R1 and R2 are the ratio arms of the bridge. They ratio the two variable resistances for current flow through the ammeter. R3 is a variable resistor known as the standard arm that is adjusted to match the unknown resistor. The sensing ammeter visually displays the current that is flowing through the bridge circuit. Analysis of the circuit shows that when R3 is adjusted so that the ammeter reads zero current, the resistance of both arms of the bridge circuit is the same. The relationship of the resistance

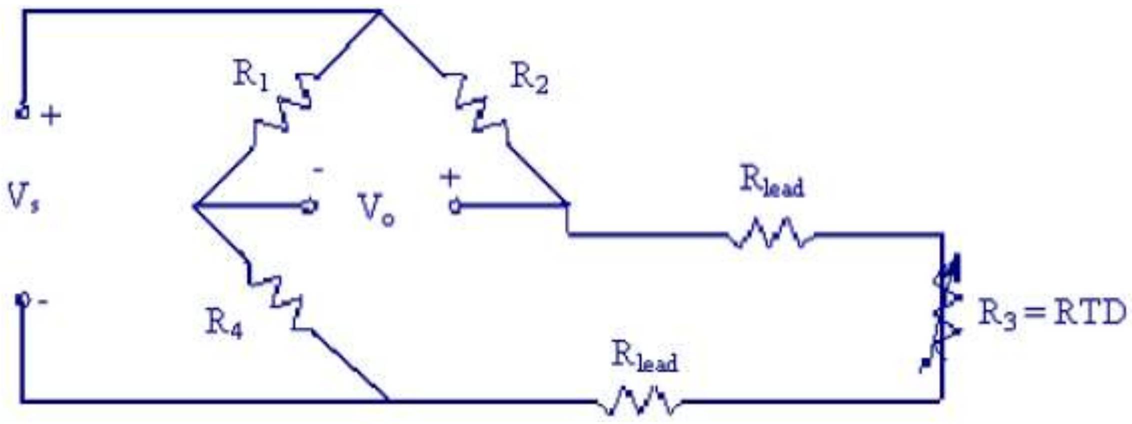
$$\frac{R_1}{R_3} = \frac{R_2}{R_x}$$

between the two arms of the bridge can be expressed asSince the values of R1, R2, and R3 are known values, the only unknown is Rx. The value of Rx can be calculated for the bridge during an ammeter zero current condition. Knowing this resistance value provides a baseline point for calibration of the instrument attached to the bridge circuit. The unknown resistance, Rx, is given by

$$R_x = \frac{R_2 R_3}{R_1}$$

RTD Bridge Circuit Operation

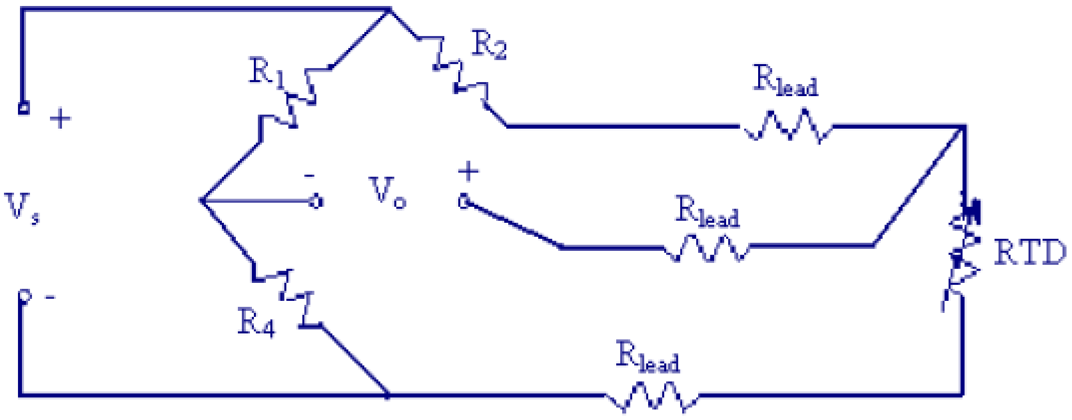
One simple circuit is the quarter bridge Wheatstone bridge circuit, here called a **two-wire RTD bridge circuit**.



R_{lead} represents the resistance of one of the wires (called lead wires) that run from the bridge to the RTD itself. Lead resistance was of no concern in strain gage circuits because R_{lead} remained constant at all times.

For RTD circuits, however, some portions of the lead wires are exposed to changing temperatures. Since the resistance of metal wire changes with temperature, R_{lead} changes with T, which can cause errors in the measurement. This error can be non-trivial – changes in lead resistance may be misinterpreted as changes in RTD resistance. Furthermore, there are two lead wires in the two-wire

RTD bridge circuit shown above, which doubles the error. A clever circuit designed to eliminate the lead wire resistance error is called a three-wire RTD bridge circuit. The three-wire RTD bridge circuit is shown below.



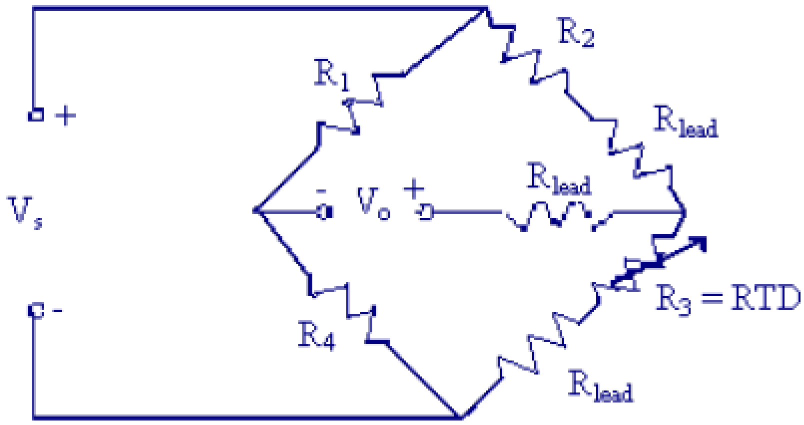
It is still a quarter bridge circuit, since only one of the four bridge resistors has been replaced by the RTD. However, one of the lead wires has been placed on the R_2 leg of the bridge instead of the R_3 leg. To analyze this circuit, assume that $R_1 = R_4$, and $R_2 = R_3$ initially, when the bridge is balanced.Recall the general formula for a Wheatstone bridge:

$$V_o = V_s \frac{R_3R_1 - R_4R_2}{(R_2 + R_3)(R_1 + R_4)}$$

Notice that R_3 and R_2 have opposite signs in the above equation. So, if the lead wire resistance in leg 2 (top) and that in leg 3 (bottom) are the same, the lead resistances cancel each other out, with no net effect on the output voltage, thus eliminating the error.

What about the third lead resistance, R_{lead} of the middle wire? Well, since V_o is measured with a nearly infinite impedance device, no current flows in the middle lead wire, so its resistance does not affect anything!

The following re-drawn equivalent circuit may help explain why the lead resistances cancel out:



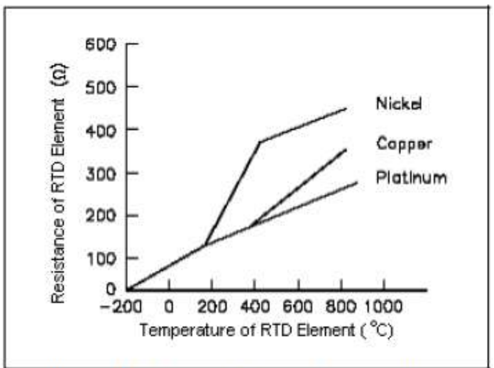
In the above diagram, it is clear that if R_{lead} changes equally in leg 2 and leg 3 of the bridge, its effect cancels out.

RTD Materials & Construction

RTD acts somewhat like an electrical transducer, converting changes in temperature to voltage signals by the measurement of resistance. The metals that are best suited for use as RTD sensors are pure metals or certain alloys of uniform quality that increase in resistance as temperature increases and conversely decrease in resistance as temperature decreases. Only a few metals have the properties necessary for use in RTD elements. Common materials used in RTD sensor are BALCO wire, Copper, Platinum.

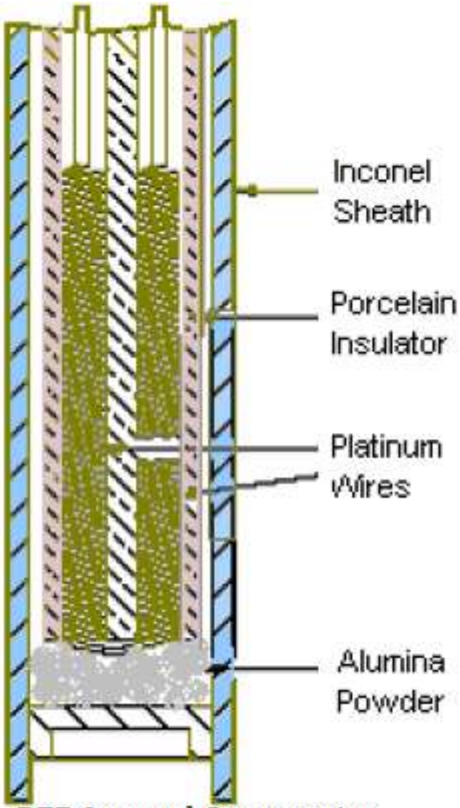
- **BALCO** – A sensor constructed using a BALCO wire is an annealed resistance alloy with a nominal composition of 70 percent nickel and 30 percent iron. A BALCO 500-ohm resistance element provides a relatively linear resistance variation from –40 to 250° The sensor is a low-mass device and responds quickly to changes in temperature. When 1000 ohms is measured across the BALCO element, the temperature is approximately 70°F. As the temperature increases, the resistance changes 2.2 ohms per 1°F. This is called a Temperature Coefficient of Resistance Curve (TCR Curve). In a BALCO, as the resistance has direct relationship with temperature i.e. as temperature increases, the resistance increases proportionally. The usual range of temperature measurement with BALCO is -40° to 240°F.
- **Platinum –** RTD sensors using platinum material exhibit linear response and stable over time. In some applications a short length of wire is used to provide a nominal resistance of 100 ohms. However, with a low resistance value, element self-heating and sensor lead wire resistance can effect the temperature indication. With a small amount of resistance change of the element, additional amplification must be used to increase the signal level. Platinum film sensor on an insulating base provides high resistance to the tune of 1000 ohms at 74° With this high resistance, the sensor is relatively immune to self-heating and responds quickly to changes in temperature. RTD elements of this type are common.

These metals are best suited for RTD applications because of their linear resistance-temperature characteristics (as shown in figure below), their high coefficient of resistance, and their ability to withstand repeated temperature cycles. The coefficient of resistance is the change in resistance per degree change in temperature, usually expressed as a percentage per degree of temperature. The material used must be capable of being drawn into fine wire so that the element can be easily constructed.



Electrical Resistance-Temperature Curves

Copper and nickel versions operate at lower temperature ranges and are less expensive than platinum. Platinum is the most versatile material because of its wide temperature range (–200°C to 850°C), excellent repeatability, stability, and resistance to chemicals and corrosion.



RTD Internal Construction

RTD elements are usually long, spring- like wires surrounded by an insulator and enclosed in a sheath of metal. Figure below shows the internal construction of an RTD.

This particular design has a platinum element that is surrounded by a porcelain insulator. The insulator prevents a short circuit between the wire and the metal sheath. Inconel, a nickel-iron-chromium alloy, is normally used in manufacturing the RTD sheath because of its inherent corrosion resistance. When placed in a liquid or gas medium, the Inconel sheath quickly reaches the temperature of the medium. The change in temperature will cause the platinum wire to heat or cool, resulting in a proportional change in resistance.

Advantages: Linear resistance with temperature, good stability, wide range of operating temperature Interchangeable over wide temperature range

Disadvantages: Small resistance change with temperature, responses may be slower, subject to self heating, transmitter or three to four wire leads required for lead resistance compensation, external circuit power required

3. INFRARED THERMOMETER

An infrared thermometer is a device that measures the infrared radiation – a type of electromagnetic radiation below the visible spectrum of light - emitted by an object. The most basic design of infrared thermometers consists of a lens to focus the infrared thermal radiation onto a detector, which converts the radiant energy into an electric signal. This configuration facilitates temperature measurement from a distance, without the need for contact with the object to be measured. The device is useful for measuring temperature under circumstances where thermocouples or other probe type sensors cannot be used.

There are many types of infrared temperature sensing devices available today, including configurations designed for flexible and portable handheld use as well as for mounting in a fixed position to serve a specific purpose.

Types of Infrared Thermometers

The most common types of infrared thermometers include:

- Spot infrared thermometers – These devices measure the temperature at a spot on a surface.
- Infrared scanning systems – These devices scan a larger area as the spot thermometer points at a rotating mirror. They are widely used in manufacturing processes involving conveyors or web processes, such as continuous piles of material along a conveyor belt or large sheets of metal or glass exiting an oven.
- Infrared thermal imaging cameras – These cameras are essentially infrared radiation thermometers used for measuring temperature at many points over a relatively large area to create a two-dimensional image called a thermogram. This technology is more software- and hardware-intensive compared to other types of infrared thermometers.

Working Principle of Infrared Thermometers

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Infrared thermometers work based on black body radiation, According to which any material with a temperature above absolute zero has molecules moving within it. The higher the temperature, the faster the molecules move. The molecules emit infrared radiation as they move, and emit more radiation, including visible light, as they get hotter. This is why a heated metal emits a red or white glow. Infrared thermometers detect and measure this radiation.

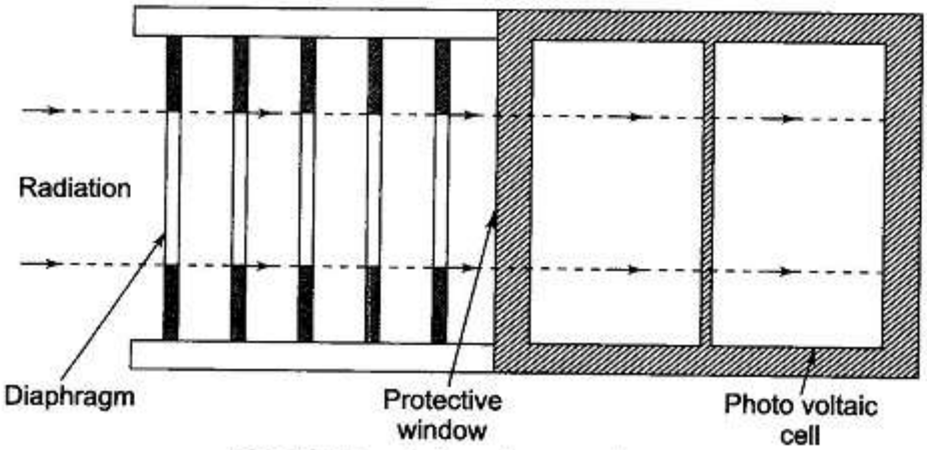

Fig.13.56 Infrared pyrometer

Infrared light can be focused, reflected or absorbed like visible light. Infrared thermometers employ a lens to focus infrared light from an object onto a detector known as a thermopile. The function of the thermopile is to absorb infrared radiation and convert it to heat. The thermopile gets hotter as it absorbs more and more infrared energy. The excess heat is converted into electricity, which is transmitted to a detector which determines the temperature of the object.

Applications of Infrared Thermometers

The major applications of infrared thermometers are given below:

- Heating and air conditioning – Detection insulation breakdown, heat loss and gain and furnace and duct leakage
- Industrial/Electrical – Monitoring motor/engine cooling systems performance, boiler operations, steam systems and detection of hot spots in electrical systems and panels
- Food safety – Checking equipment performance, sanitation and process temperature conditions, and scanning refrigerated display cases, trucks, storage areas and cooling systems
- Agriculture – Monitoring plant temperatures for stress and animal bedding to detect spoiling.